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Final Research Report

Project #: UA13-DLA

Mitigating the Impact of Lead Time Variability

Industry Partner: *Defense Logistics Agency*

Mitigating the Impact of Lead Time Variability

Research Team: Manuel D. Rossetti (PI), Juliana Bright (RA)

Sponsor: **Defense Logistics Agency**	***Thrust Area:*** **Inventory**

Keywords: lead-time, reorder point reorder quantity policies, reduced models

Problem in Context: The main goal of this research is to model the effect of lead-time variability on the DLA's logistics system and to develop methods to incorporate this information into inventory policy setting and planning. The key parameters for inventory policy setting include characterizations of lead-time, demand, and costs (e.g. holding, ordering, back ordering, etc.). The information on the lead-time and the demand is combined into a model of demand during lead-time and used in inventory stocking models. These models enable the setting of safety stock values in order to minimize cost at acceptable service levels. Current models used within the DLA do not adequately account for the variability in lead-time. In addition, methods for how to characterize the lead-time (i.e. estimate mean values, variances, 85th percentile, etc.) and incorporate measures of uncertainty in lead-time into planning models are not well understood. This research will investigate how to properly incorporate lead-time variability into DLA inventory planning systems by analyzing current methods, developing new models, and comparing the impact of the models on the cost, availability, and operational performance of the DLA's logistics systems.

Technical Approach: This project examines how cost and other inventory control parameters in an (r, Q) inventory control policy mitigate or exacerbate the impact of increasing lead time variability on system performance using a computational experiment drawn from industrial datasets. The standard single-item single-location stochastic *(r, Q)* inventory model is compared with models that ignore lead time variance and adjust other lead time and demand parameters to protect against the deleterious effects of high lead time variability.

Results: A software tool that implements two of the adjusted models was developed in Java. The best results were found when the demand variance was inflated instead of inflating the constant lead time.

Broader Value to CELDi Members: This proposed research has the broader impact of examining the best way to compensate for the erroneous assumption of constant lead time.

Future Research and Potential Extensions: Time-phased demand

Project # UA11-INVS. **February 19, 2016**

Mitigating the Impact of Lead Time Variability

Manuel D. Rossetti, Ph. D., P. E. (Principal Investigator),

Juliana Bright (Research Assistant)

This report is intended for distribution within the

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Mitigating the Impact of Lead Time Variability

Manuel D. Rossetti , Ph. D., P. E., Juliana Bright
Department of Industrial Engineering
University of Arkansas
4207 Bell Engineering Center

Fayetteville, AR 72701

Executive Summary

The main goal of this research is to model the effect of lead-time variability on the DLA's logistics system and to develop methods to incorporate this information into inventory policy setting and planning. The key parameters for inventory policy setting include characterizations of lead-time, demand, and costs (e.g. holding, ordering, back ordering, etc.). The information on the lead-time and the demand is combined into a model of demand during lead-time and used in inventory stocking models. These models enable the setting of safety stock values in order to minimize cost at acceptable service levels. Current models used within the DLA do not adequately account for the variability in lead-time. This leads to inadequate safety stock and customer service levels that fall short of planned targets.

This report compares several LTD models for use in an *(r, Q)* inventory policy: LTD*, LTD_V, LTD_0, LTD_{CV}, and $LTD_{L.}$ LTD* is the best-practice model for *(r, Q)* inventory policies. The variance inflation model (LTD_V or VIM) always gives the same results as LTD* (total annual cost $13.5 million and no service level deficits) and is simple to implement.

Further research and modeling is needed to determine the best way to model items without historical data, such as new items. LTD_{CV}, estimating lead time variance as 30% of the mean, had a total annual cost of $10.0 million dollars and average service level deficit of 1.70%. These numbers do not represent a substantial improvement over LTD_0, the model where lead time variance was ignored completely, which had a total annual cost of $9.97 million and an average service level deficit of 1.74%. Linear regression was attempted over the test dataset, but the fitted models violated standard regression assumptions and had low R^2 values as well (ranging from 0.2514 to 0.5075). Therefore, straightforward linear regression modeling is likely to not be successful in predicting lead time variance based on the provided datasets.

While LTD_L, inflating the lead time mean but neglecting lead time variance, was an improvement over the current model LTD_{CV}, with total annual cost of $13.9 million and a

service level deficit of only 0.684%, it requires the same historical data as LTD* or LTD_V but is more computationally intensive to use and less accurate. If historical data is available but the inventory system assumes lead time is constant, LTD_V is simpler to implement and more accurate than LTD_L and is therefore the recommended alternative. If the inventory system allows for stochastic lead time, then the lead time variance should be collected from historical data if possible and LTD* used.

Recommendations

The key findings and recommendations for this research include:

- If the inventory system allows for stochastic lead time, then the lead time variance should be collected from historical data if possible and LTD* used.
- If historical data is available but the inventory system assumes lead time is constant, LTD_V should be used.
- If historical data is not available, further research is needed. Using 30% of mean lead time as point estimate for lead time variance does not give much improvement over treating lead time as constant.

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1 INTRODUCTION

The main goal of this research is to model the effect of lead-time variability on the DLA's logistics system and to develop methods to incorporate this information into inventory policy setting and planning. The key parameters for inventory policy setting include characterizations of lead-time, demand, and costs (e.g. holding, ordering, back ordering, etc.). The information on the lead-time and the demand is combined into a model of demand during lead-time and used in inventory stocking models. These models enable the setting of safety stock values in order to minimize cost at acceptable service levels. Current models used within the DLA do not adequately account for the variability in lead-time. This leads to inadequate safety stock and customer service levels that fall short of planned targets.

In addition, methods for how to characterize the lead-time (i.e. estimate mean values, variances, 85^{th} percentile, etc.) and incorporate measures of uncertainty in lead-time into planning models are not well understood. This research will investigate how to properly incorporate lead-time variability into DLA inventory planning systems by analyzing current methods, developing new models, and comparing the impact of the models on the cost, availability, and operational performance of the DLA's logistics systems.

2 PREVIOUS WORK

Lead time variability has been analyzed for 40 years in the academic literature, indicating that it is an important factor in an inventory system. The earliest paper reviewed for this project was Vinson (1972), which examined the then-current assumption that lead time was constant. The paper analyzed stock outs and safety stocks in (r, Q) inventory model and found that holding demand process constant and increasing lead time variability had a much bigger impact on stock outs and safety stocks than holding lead time process constant and increasing demand variability, suggesting that treating stochastic lead time as constant was a dangerous assumption.

This finding was also confirmed by Bagchi et. al. (1986), which found that mean period demand and lead time variance are biggest contributors to safety stock. Their analysis found that assuming a constant lead time estimated using the mean radically underestimated the variance of lead time demand. In consequence, safety stocks and service levels were set too low, leading to underestimating the probability of stock outs. To ameliorate this problem, the paper suggested using a "safety lead time" that was higher than the mean. For example, they suggested choosing a percentile of the lead time distribution that was high enough to meet service level targets. This report formalizes the notion of "safety lead time" in *(r, Q)* model and investigates its efficacy compared to other ways of handling lead time variance.

Moreover, Eppen and Martin (1987) found that the lead time demad (LTD) distribution family also mattered. Specifically, safety stock is set too low when LTD is assumed normal. They also showed that, contrary to expectation, the central limit theorem does not imply that lead time demand is normally distributed. Therefore, setting safety stock based on normal distribution underestimates probability of stock out. Instead, they derive the variance of cumulative error of a forecast using simple exponential smoothing and use the results to solve for safety stock in deterministic and stochastic lead time cases. This report therefore does not fit a normal distribution to LTD.

Paknejad et. al. (1992) studied *(r,Q)* inventory models with stochastic demand and stochastic lead times. The paper presents a nonlinear optimization model for determining how much to invest decreasing lead time variability. The model requires formulating an ROI function that gives the cost of decreasing lead time. This helps deriving the exact solution when the ROI function is logarithmic, corresponding to decreasing marginal return on investment. We borrowed the nonlinear optimization approach in defining an optimal constant lead time estimate.

Moon and Gallego (1994) use a minimax optimization approach. An upper bound on the first order loss function for any distribution is derived based on the first two moments. The optimality conditions for minimizing the cost of an *(r, Q)* policy are then derived based on the bound, which can be the basis of a nonlinear optimization algorithm. The cost of the resulting policy can be compared to the cost of the optimal policy found using the "true" distribution (e.g. a Gamma distribution), which the authors call the "expected value of additional information" (EVAI). We are using their EVAI to compare inventory models and expanding it.

Song (1994) provided some theoretical groundwork that showed how increasing lead time variability always increases total cost of a base stock model. The paper compared two base stock models that were identical except for the lead time random variables (called L1 and L2) using stochastic dominance. The paper considered two cases. In the first, L1 is has a smaller mean but a greater variance than L2. The paper showed that it can be more cost effective to use L2 even though the base stock level is higher, giving the example of Poisson demand and exponential lead time. In the second case, L1 is less variable than L2. In this situation, lead time demand is more varies under L2 than L1. As a result, it is always more cost effective to use L1 although the optimal base stock level can be higher. Since *(r, Q)* models can be considered as combinations of base stock models, Song's analysis also provides theoretical grounding for concern about the impact of lead time variance.

Dolgui and Ould-Louly (2002) is a possibly different way to estimate safety lead time. In their paper, they examined planned order releases in an MRP system. MRP planned order releases are modeled as base-stock inventory policy with constant demand and discrete stochastic lead time. The model distribution is derived as a Markov Chain. The optimization model focuses on the time an order should be released. The result states that the time an order should be released is based on a percentile of lead time distribution (T) and the number of periods before deadline to release an order. This model may be of interest in the current, independent demand situation.

Finally, So and Zheng (2003) look at lead time, capacity, and forecasting in a system with stochastic demand and constant lead time in a two-level system. This could correspond to the contract negotiation-warehouse system at DLA. They show that, as demand variability increases at the retailer, backlogs build up at the supplier and result in much larger lead times, and autocorrelated demand and capacity utilization at the supplier multiply the effects. They conclude that contracts whose demands are highly variable and autocorrelated should be prioritized. This suggests an interesting avenue for further exploration and also suggests that DLA should perhaps monitor items whose demands are highly variable and autocorrelated more closely.

3 THE STANDARD (r, Q) INVENTORY MODEL

In this section the standard *(r, Q)* single-item stochastic inventory policy is presented. Input parameters, including lead time variance, and metrics to evaluate policy performance are discussed first. Next are methods to find optimal reorder point and size. The section concludes by introducing an example item and demonstrating how to set up, calculate performance, and optimize the example item.

The *(r, Q)* inventory policy is a continuous-review policy, in which an order for Q items is placed whenever the inventory position falls at or below the reorder point r. In addition to r and Q, the policy also requires a fitted probability distribution model of demand during lead time, called the LTD model, cost information, and/or a a service level constraint γ (on fill rate). The LTD model is built from the moments of the demand distribution (μ_D, σ^2_D) and lead time distribution (μ_L, σ^2_L). Costs needed are ordering cost K in \$/order and holding cost h in \$/item/period. Finally, a backorder cost b in \$/item/period or a fill rate constraint γ are needed to control backorder and customer service. This is summarized in Table 1.

Table 1: Parameters of *(r, Q)* models

Parameter Group	Parameter	Name
Policy	r	Reorder point
	Q	Reorder quantity
	γ	Service level
LTD Model	μ_D	Mean demand
	σ^2_D	Variance of demand
	μ_L	Mean lead time
	σ^2_L	Variance of lead time
Costs	K	Ordering cost ($/order)
	c	Item cost ($/unit)
	i	Holding charge ($/$/time)
	$h=ic$	Holding cost ($/unit/time)
	b	Backorder cost ($/unit/time)

The demand and lead time distributions are combined to derive the distribution of demand during replenishment lead time (LTD distribution) as illustrated in Figure 1. The moments $(\mu_D, \sigma_D^2, \mu_L, \sigma_L^2)$ of demand and lead time are combined using:

$$\mu_{LTD} = \mu_D \mu_L$$
$$\sigma_{LTD}^2 = \mu_L \sigma_D^2 + \mu_D^2 \sigma_L^2$$

A distribution is fit to these moments. This project used the Gamma distribution, because it has been shown to work better than the Normal (Rossetti & Unlu, 2011). A Gamma distribution is fit with parameters α and β using the following equations:

$$\alpha = \frac{\mu_{LTD}^2}{\sigma_{LTD}^2}$$
$$\beta = \frac{\sigma_{LTD}^2}{\mu_{LTD}}$$

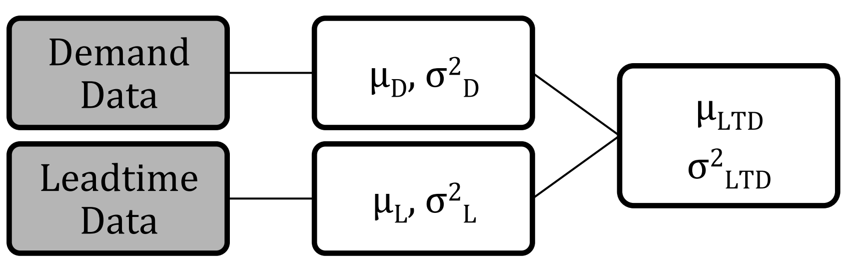

Figure 1: Relationship between the demand, lead time, and LTD moments.

The economic order quantity (EOQ) is a well-known lower bound on Q^*. Q_{LB} is a tighter lower bound for stochastic inventory models. The equations are below.

$$EOQ = \sqrt{\frac{2\mu_D K}{h}}$$

$$Q_{LB} = \sqrt{\frac{2\mu_D K}{h\gamma}}$$

3.1 MEASURING THE PERFORMANCE OF AN *(r, Q)* POLICY

Many performance measures for an inventory policy have been devised. All of the performance measures represent long-run expected values. This spreadsheet uses some common performance measures: inventory on hand, backorders outstanding, ready rate (basically, the fill rate), order frequency, safety stock, total cost per period and its components ordering cost, holding cost, and backorder cost, the value of inventory on hand, the value of safety stock, and annual operating cost. The performance measures and their meanings are summarized in Table 2.

Table 2: Important inventory performance measures

Symbol	Formula	Name	Interpretation
I	$\bar{I}(r,Q\|LTD_*) = \frac{Q+1}{2} + r - \mu_{LTD} + \bar{B}(r,Q\|LTD_*)$	On hand	Average number of units in stock
B	$\bar{B}(r,Q\|LTD_*) = \frac{1}{Q}\left(F^2_{LTD}(r) - F^2_{LTD}(r+Q)\right)$	Backorders	Average number of units on backorder
RR	$\overline{RR}(r,Q\|LTD_*) = 1 - \frac{F^1_{LTD}(r) - F^1_{LTD}(r+Q)}{Q}$	Ready rate	The probability that no stock is on hand when demand occurs; sometimes called fill rate.
OF	$\overline{OF}(r,Q\|LTD_*) = \frac{\mu_D}{Q}$	Order frequency	Average number of orders placed per period
SS	$\overline{SS}(r,Q\|LTD_*) = \max\,(\,r - \mu_{LTD}, 0)$	Safety stock	Number of units expected to be in stock when a replenishment order arrives
C	$C(r,Q\|LTD_*) = K\overline{OF} + h\bar{I}$	Relevant cost	Average total inventory cost per period, as calculated at DLA.
LC	$LC(r,Q\|LTD_*) = K\overline{OF} + h\bar{I} + b\bar{B}$	Lagrangian cost	Relevant cost plus backorder costs, which are often a Lagrangian multiplier for the service level.

3.2 Finding the Optimal *(r, Q)* Policy

The cost function for an (r, Q) inventory control model is given by:

$$C(r,Q|LTD) = \frac{K\mu_D}{Q} + h\left(\frac{Q+1}{2} + r - \mu_{LTD} + \frac{F^2_{LTD}(r) - F^2_{LTD}(r+Q)}{Q}\right)$$

In this equation, $F^2_{LTD}(.)$ is the second-order loss function of the LTD distribution, which was fit as described in the previous section. The *(r, Q)* policy may now be optimized. Since this spreadsheet is using a continuous probability distribution, any nonlinear optimization method (even Excel's Solver) can be used to find the *r* and *Q* that result in the lowest total cost by solving the following problem:

$$\begin{gathered}\min C(r,Q|LTD)\\ \text{subject to}\\ RR(r,Q|LTD) \geq \gamma\\ r \geq -Q\\ Q \geq 0\end{gathered}$$

This model is may solved using Lagrangian optimization by adding a penalty cost b for backorders. The fill rate of the optimal policy, called *(r*, Q*)*, is critical ratio $\omega = \frac{b}{h+b}$. To meet the target service level (i.e. ensure that the ready rate will be γ), the backorder cost b is treated as a Lagrangian multiplier and calculated from the holding cost and service level using the following equation:

$$b = \frac{\gamma h}{1-\gamma}$$

After simplification, the objective function of the Lagrangian problem is:

$$LC(r, Q|LTD) = \frac{K\mu_D}{Q} + h\left(\frac{Q+1}{2} + r - \mu_{LTD}\right) + \frac{h}{1-\gamma}\left(\frac{F^2_{LTD}(r) - F^2_{LTD}(r+Q)}{Q}\right)$$

The Lagrangian problem is given by the following nonlinear program.

$$\begin{gathered}\min C(r, Q|LTD)\\ \text{subject to}\\ r \geq -Q\\ Q \geq 0\end{gathered}$$

3.3 Example Item

The previous section presented the standard single item stochastic *(r, Q)* inventory model. This section illustrates the standard *(r, Q)* inventory policy using an example SKU. The same SKU will also be used throughout the remainder of this report to illustrate the relationship between the standard *(r, Q)* inventory policy and variants examined during the project. Table 3 summarizes the necessary parameters and gives the data used in the example.

The standard *(r, Q)* policy constructs the LTD distribution from all four input moments $(\mu_D, \sigma_D^2, \mu_L, \sigma_L^2)$. This LTD model will be referred the full model or LTD_* below, since it will be contrasted with some simplified or reduced models further on in this report.

- **Table 3: Inputs to (r, Q) model and example data**

Input Type	Symbol	Name	Example Data
Time units	periods	Periods or time buckets	days
Cost/service	K	Ordering cost	\$5/order
	c	Item cost	\$100
	i	Holding charge per item per period	0.0025
	γ	Target fill rate (customer service)	0.95
Random variables	μ_D	Mean demand per period	10
	σ_D	Standard deviation of demand per period	2
	μ_L	Mean lead time in periods	14
	σ_L	Standard deviation of lead time in periods	3

First, the holding and backorder costs are calculated.

$$h = ic = 0.0025 * 100$$
$$h = \$0.25/item/day$$

$$b = \frac{\gamma h}{1-\gamma} = \frac{0.95 * 0.25}{0.05}$$
$$b = \$4.75/item/day$$

Next the LTD distribution is fit.

$$\mu_{LTD} = \mu_D \mu_L = 10 * 14$$
$$\mu_{LTD} = 140 \frac{items}{day}$$
$$\sigma_{LTD}^2 = \mu_L \sigma_D^2 + \mu_D^2 \sigma_L^2$$
$$\sigma_{LTD}^2 = 14 * 2^2 + 10^2 * 3^2$$
$$\sigma_{LTD}^2 = 956$$

$$\alpha = \frac{\mu_{LTD}^2}{\sigma_{LTD}^2} = \frac{140^2}{956}$$
$$\alpha = 20.5$$
$$\beta = \frac{\sigma_{LTD}^2}{\mu_{LTD}} = \frac{956}{140}$$
$$\beta = 6.83$$

$$LTD_* \sim Gamma(20.5, 6.83)$$

The Lagrangian objective function is

$$LC(r, Q|LTD) = \frac{50}{Q} + 0.25\left(\frac{Q+1}{2} + r - 140\right) + 5\left(\frac{F_{LTD}^2(r) - F_{LTD}^2(r+Q)}{Q}\right)$$

The optimal policy is (178.79, 36.215). The annual cost is $5,774.72, of which ordering costs comprise $503.93 and holding costs $5,270.79. The ready rate is 95%, meeting the service level requirement.

4 REDUCED LTD MODEL

In this section, the standard *(r, Q)* policy is contrasted a reduced model that assumes lead time is constant. The impact of this assumption on the optimal policy of the example item is illustrated. The results suggest new ways of measuring the performance of the reduced model. Finally, ways of improving the reduced model are examined.

One common simplification is to assume that lead time is constant. The inventory models that make this assumption are called reduced models in this document to contrast them with the full model. The reduced models examined in this project only use three of the four moments available, (μ_D, σ^2_D, μ_L), to fit an LTD distribution. The reduced model will be referred to as LTD_0 throughout the report. The reduced model is equivalent to using the parameters (μ_D, σ^2_D, μ_L, 0) for the full model. As a result, the moment matching equations become:

$$\mu_{LTD} = \mu_L \mu_D$$
$$\sigma^2_{LTD} = \mu_L \sigma^2_D$$

Clearly, then, the LTD variance is much smaller in the reduced model, so less safety stock is required to meet a given service level compared with the full model. Conversely, when the full model is more accurate (that is, if lead time variance is non-trivial), the reorder point/safety stock levels suggested by the reduced model will be inadequate to meet the service level requirements. It should also be noted that the minimum-cost or optimal policy *(r^*_0, Q^*_0)* of the reduced policy LTD_0 will differ from the optimal policy *(r^*, Q^*)* of the full policy LTD^*.

4.1.1 EXAMPLE

In this section, the reduced model corresponding to the full model of the previous section is presented. The mean of the reduced model is still 140 units/day, but the variance is only 56 units^2/day^2 instead of 956 in the full model. This time, the parameters of the Gamma distribution of LTD are:

$$\alpha = \frac{\mu_{LTD}^2}{\sigma_{LTD}^2} = \frac{140^2}{56}$$
$$\alpha = 350$$
$$\beta = \frac{\sigma_{LTD}^2}{\mu_{LTD}} = \frac{56}{140}$$
$$\beta = 0.4$$

$$LTD_0 \sim Gamma(350, 0.4)$$

The Lagrangian objective function has the same coefficients as before; the difference lies in the second order loss function, $F_{LTD}^2(\cdot)$.

$$LC(r, Q|LTD) = \frac{50}{Q} + 0.25\left(\frac{Q+1}{2} + r - 140\right) + 5\left(\frac{F_{LTD}^2(r) - F_{LTD}^2(r+Q)}{Q}\right)$$

The optimal policy (r^*_0, Q^*_0) of the reduced model is (144.75, 24.369). The annual cost is \$2,312.54, of which ordering costs comprise \$748.92 and holding costs \$1,563.63. The ready rate is still 95%, meeting the service level requirement. On the surface this policy looks much better than the full model, but the next section discusses the problems with using the reduced model.

4.1.2 Performance Metrics for Reduced Models

Performance metrics for a reduced model are more complicated than those of the full model. In general, differences between LTD_0 and LTD_* will lead to different optimal policies (r_0^*, Q_0^*) and (r^*, Q^*) with different performance. Basically, each performance measure has two kinds of inputs: the policy, that is, the *r* and *Q*, and the LTD model, e.g. LTD_* or LTD_0. In particular, the performance of the reduced model optimal policy (r^*_0, Q^*_0) may differ greatly depending on whether it is evaluated using LTD* or LTD_0.

Using relevant cost and ready rate as the example performance measures, we can define three different performance measures to use in comparing the full model to the reduced model. This applies to each performance measure in Table 2, but will be illustrated using the two most important metrics, relevant cost and ready rate.

1. Best Possible Performance: This is the performance of the optimal policy of the full model. This is the "best" policy of the "best" LTD model: $C(r^*, Q^* \mid LTD_*), RR(r^*, Q^* \mid LTD_*)$

2. Expected Performance: This is the performance of the optimal policy of the reduced model. In other words, this corresponds to the predictions of the reduced model: $C(r_0^*, Q_0^* \mid LTD_0), RR(r_0^*, Q_0^* \mid LTD_0)$

3. Realized Performance: But we are assuming that lead time variance matters – that is, that LTD* is a more accurate model of demand during lead time than LTD_0. So while the minimum cost policy (r^*_0, Q^*_0) can be computed for LTD_0, the long run average performance realized in practice should be evaluated according to LTD*: $C(r_0^*, Q_0^* \mid LTD_*), RR(r_0^*, Q_0^* \mid LTD_*)$

4.1.2.1 EXAMPLE

In hindsight, therefore, the policy and performance computed in section 3.3 corresponds to "Best" and the performance computed in section 3.4.1 corresponds to "Expected". The "Realized" performance of the optimal policy of the reduced model is given by

$$C(144.75, 24.369|LTD_*) = \$2{,}868.66$$

$$RR(144.75, 24.369|LTD_*) = 72.0\%$$

where $LTD_* \sim Gamma(20.5, 6.83)$.

The table below summarizes the three performance measures used to compare a reduced model to the corresponding full model.

Table 4

	LTD MODEL	
OPT. POLICY	**Full** $\boldsymbol{LTD_* \sim Gamma(20.5, 6.83)}$	**Reduced** $\boldsymbol{LTD_0 \sim Gamma(350, 0.4)}$
Full $(\boldsymbol{r}^*, \boldsymbol{Q}^*)$ $(\mathbf{178.79, 36.215})$	Best $C(r^*, Q^*\|LTD_*) = \$5{,}774.72$ $RR(r^*, Q^*\|LTD_*) = 95.0\%$	Solution-to-Solution $C(r^*, Q^*\|LTD_0) = \$5{,}695.86$ $RR(r^*, Q^*\|LTD_0) > 99.99\%$
Reduced $(\boldsymbol{r_0^*}, \boldsymbol{Q_0^*})$ $(\mathbf{144.75, 24.369})$	Realized $C(r_0^*, Q_0^*\|LTD_*) = \$2{,}868.66$ $RR(r_0^*, Q_0^*\|LTD_*) = 72.0\%$	Expected $C(r_0^*, Q_0^*\|LTD_0) = \$2{,}312.54$ $RR(r_0^*, Q_0^*\|LTD_0) = 95.0\%$

While it is true that the realized cost of (r^*_0, Q^*_0) is much less than (r^*, Q^*) – \$2,868.66 vs. \$5,774.72, this reduction in cost comes at the expense of the realized service level, which drops from 95% to 72%. Using LTD_0 to set the (r, Q) levels will result in a system that consistently falls short of its customer service targets; in particular, the costs occurred in practice will generally be higher than expected, the ready rate will fall short, and stockouts will occur much more frequently than expected.

4.2 ADJUSTING REDUCED MODEL

In this section, a number of ways to adjust or improve LTD_0 are discussed. The model currently in use, which uses a point estimate of the variance mean ratio, is discussed first;

then adjustments based on moment-matching are proposed. Each adjusted model is illustrated using the example item and some basic contrasts are pointed out.

4.2.1 CV Estimate Model

This model attempts to improve over the reduced model LTD_0 by estimating σ_L^2 as $\alpha * \mu_L$ for some real number α, which corresponds to the variance mean ratio (VMR) of lead time. This model will be called LTD_α. DLA currently uses this model with $\alpha = 0.3$ for all SKUs.

This model is equivalent to using parameters $(\mu_D, \sigma_D^2, \mu_L, \alpha\mu_L)$ in the full model. The accuracy of this model will depend on how accurate α is as an estimate of the VMR. Assuming that the lead time is constant, as in LTD_0, is equivalent to setting α=0. If α is allowed to vary for each SKU, then the optimal α for each SKU is clearly the VMR.

4.2.1.1 Example

The CV adjustment model will be illustrated using the example item with α=0.3. For the example item, the LTD distribution is fit via:

$$\mu_{LTD} = \mu_D\mu_L = 10 * 14 = 140$$
$$\sigma_{LTD}^2 = \mu_L\sigma_D^2 + \alpha\mu_D^2\mu_L = 14 * 4 + 0.3 * 10^2 * 14 = 476$$
$$LTD{\sim}Gamma(41.2, 0.294)$$

The optimal policy for LTD_{CV} is (164.49, 32.068) and the expected annual relevant cost is \$4,319.58, close to the realized cost of \$4,455.79. Both the realized and expected costs are less than LTD_*'s optimal cost of \$5,774.72. This reduction in cost, however, comes as the expense of the realized service level (89.1%), which is 5.9% below target. The performance of this policy is summarized in Table 5. The model's estimate of annual relevant cost is not too far off the realized cost but it overestimates the ready rate by 5.9%.

Table 5: Performance Characteristics of LTD*, LTD_0, and LTD_{CV}

	LTD*	**LTD_0**	**LTD_L**
r	178.8	144.8	164.49
Q	36.2	24.4	32.068
***C* (realized)**	\$5,774.72	\$2,868.66	\$4,455.79
***RR* (realized)**	95.0%	72.0%	89.1%
***C* (expected)**		\$2,312.54	\$4,319.58
***RR* (expected)**		95.0%	95.0%

4.2.2 Moment Matching Models

As seen above, simply assuming that the lead time is constant in the reduced model leads to underestimating the cost and overestimating the ready rate compared to the expected realizations in operation. On the other hand, fitting a LTD model only requires the combined moments $(\mu_{LTD}, \sigma_{LTD}^2)$, so the LTD distribution only depends indirectly on the demand and lead time moments $(\mu_D, \sigma_D^2, \mu_L, \sigma_L^2)$. This suggests an method to improve the reduced model: match the LTD moments of the reduced model to the full model by adjusting the input moment parameters from $(\mu_D, \sigma_D^2, \mu_L, \sigma_L^2)$ to some combination of $(D, V, L, 0)$ respectively. The generic equations are listed below.

$$\mu_{LTD} = \mu_D \mu_L = DL$$
$$\sigma_{LTD}^2 = \mu_L \sigma_D^2 + \mu_D^2 \sigma_L^2 = LV$$

Several possible solutions to the system of equations above will be discussed in this section. The first is to adjust the (constant) lead time from μ_L to some L so that the input parameters to the LTD model are $(\mu_D, \sigma_D^2, L, 0)$. The second is to adjust both the mean demand from μ_D to some D and the lead time from μ_L to some L so that the input parameters to the LTD model are $(D, \sigma_D^2, L, 0)$. The final approach is to adjust the σ_D^2 so that the input parameters to the LTD model are $(\mu_D, V, \mu_L, 0)$.

4.2.3 Mean Inflation Model

The most obvious adjustment to make is to inflate the lead time L to allow for a buffer, much as safety stock buffers customer demand. This model is called LTD_L and is similar to the "safety lead time" approach used in Bagchi et. al (1986). This has also been called the "percentile solution" when L is set to some percentile of the random lead time.

$$\mu_D \mu_L = \mu_D L$$
$$\mu_L \sigma_D^2 + \mu_D^2 \sigma_L^2 = L \sigma_D^2$$

The moment matching equations are above. This system has two equations and one unknown, so the system is inconsistent. In other words, LTD_L will always be different from LTD_*. On the other hand, it is possible to find some least-squares solution L^*. Since relevant cost and service level are the most important metrics, they are summarized by the mean relative squared error (MSRE). The optimal value for L, called L^*, is found by solving a bilevel mathematical program to minimize the MSRE of best vs. realized.

4.2.3.1 Example

Returning to the example item, the optimal value to use for the constant lead time, L^*, was found by solving the mathematical program mentioned above. The result is a lead time of 17.5 days (Table 6). The lead time is assumed to have a Gamma distribution. 17.5 days is the 87.7th percentile. The realized performance of LTD_L is much closer to that of LTD*: cost is only 3.9% below LTD* and the ready rate is only 1.1% less.

The expected performance is not quite as accurate, though. The expected cost is $3,291.79 less than the realized cost and the expected ready rate is 0.9% below the realized ready rate. Compared to LTD_0, the realized performance is much closer to LTD*. In other words, the mean inflation model sacrifices prediction accuracy for operational accuracy.

Table 6: Performance Characteristics of LTD*, LTD_0, and LTD_L

	LTD*	LTD_0	LTD_L
L*	14	14	17.5 (87.7th percentile)
r	178.8	144.8	179.3
Q	36.2	24.4	24.8
***C* (realized)**	$5,774.72	$2,868.66	$5,552.93
***RR* (realized)**	95.0%	72.0%	93.9%
***C* (expected)**		$2,312.54	$2,261.14
***RR* (expected)**		95.0%	93.0%

4.2.4 Joint Mean Adjustment Model

While the system of moment matching equations is inconsistent when only one parameter (the lead time L) is varied, an exact solution exists when the two mean parameters, D and L, are varied simultaneously. This model is called LTD_{LD}. The solution to the moment matching equations is:

$$D = \frac{\mu_D \mu_L \sigma_D^2}{\mu_L \sigma_D^2 + \mu_D^2 \sigma_L^2} = \frac{\sigma_D^2 \mu_{LTD}}{\sigma_{LTD}^2}$$
$$L = \mu_L + \frac{\mu_D^2 \sigma_L^2}{\sigma_D^2} = \frac{\sigma_{LTD}^2}{\sigma_D^2}$$

Unfortunately, the ordering cost is DK/Q, so the first term of the cost function $C(r, Q \mid LTD_{LD})$ will be different from $C(r, Q \mid LTD^*)$, but the other terms will be identical. Therefore, this model will also result in a slightly different optimal policy than LTD*.

It is true that LTD_{LD} can be further improved by increasing the ordering cost to K‘ using the equation below, but Occam's Razor suggests trying a different approach first.

$$K' = \frac{\mu_D K}{D} = \frac{\mu_D K \sigma_{LTD}^2}{\sigma_D^2 \mu_{LTD}} = \frac{\mu_D K(\mu_L \sigma_D^2 + \mu_D^2 \sigma_L^2)}{\mu_D \mu_L \sigma_D^2}$$

4.2.4.1 Example

The LTD distribution for the example item has a mean of 140 and a variance of 956. Therefore, the D and L values are:

$$D = \frac{4 * 140}{956} = 0.586$$

$$L = \frac{956}{4} = 239$$

To match the moments, D is very low and L is very high. Because the mean demand decreased so much, the optimal policy (187.9, 13.8) has a slightly higher reorder point but a much lower reorder quantity. The realized ready rate of LTD_{LD}, 95% , hits the service level target, but the cost is 10.8% higher than LTD*. Indeed, absolute error in the realized cost of LTD_{LD} is greater than LTD_L (respectively \$622.94 vs. \$221.79).

The expected performance of LTD_{LD} is much more accurate than LTD_L. The expected cost is \$3,291.79 less than the realized cost and the expected ready rate is 0.9% below the realized ready rate. Compared to LTD_0, the realized performance is much closer to LTD*, but compared to LTDL, the realized performance is worse.

Table 7: Performance Characteristics of LTD*, LTD_0, LTD_L, and LTD_{LD}

	LTD*	LTD_0	LTD_L	LTD_{LD}
D	10	10	10	0.586
L	14	14	17.5	239
r	178.8	144.8	179.3	187.9
Q	36.2	24.4	24.8	13.8
***C* (realized)**	\$5,774.72	\$2,868.66	\$5,552.93	\$6,397.66
***RR* (realized)**	95.0%	72.0%	93.9%	95.0%
***C* (expected)**		\$2,312.54	\$2,261.14	\$5,154.99
***RR* (expected)**		95.0%	93.0%	95.0%

4.2.5 VARIANCE INFLATION SOLUTION

Since the main impact of assuming lead time is constant is on the LTD variance, it makes sense to leave the means μ_D and μ_L alone and concentrate on matching the variance by itself. In this case the moment matching equations are:

$$\mu_{LTD} = \mu_D \mu_L$$
$$\sigma_{LTD}^2 = \mu_L \sigma_D^2 + \mu_D^2 \sigma_L^2 = \mu_L V$$

The means match, so only the second equation must be solved. The inflated demand variance V is therefore given by:

$$V = \sigma_D^2 + \frac{\mu_D^2 \sigma_L^2}{\mu_L} = \frac{\sigma_{LTD}^2}{\mu_L}$$

When using this model, called LTD_V, none of the coefficients of the cost equation are affected, unlike the previous model, LTD_{LD}. Moreover, none of the performance measures used in this project include σ_D^2 as a coefficient. Therefore, LTD_V has the exact functional form as LTD* and will have the same optimal policy and predict the same performance measures. For LTD_V, there is no difference between expected and realized performance; both are the same as the best possible performance *(r*, Q*, LTD*).*

4.2.5.1 EXAMPLE

Returning to the example item, the parameters for LTD_V can be computed.

$$V = \frac{\sigma^2_{LTD}}{\mu_L} = \frac{956}{14} = 68\frac{2}{7}$$

$$\mu_{LTD} = 10 * 14 = 140$$

$$\sigma^2_{LTD} = 14 * \frac{956}{14} = 956$$

Fitting the LTD distribution produces the same results as LTD*.

$$\alpha = \frac{\mu^2_{LTD}}{\sigma^2_{LTD}} = \frac{140^2}{956}$$

$$\alpha = 20.5$$

$$\beta = \frac{\sigma^2_{LTD}}{\mu_{LTD}} = \frac{956}{140}$$

$$\beta = 6.83$$

$$LTD_V \sim Gamma(20.5, 6.83)$$

The optimal policy for LTD_V is (178.79, 36.215), the same as LTD*. The annual cost is $5,774.72, of which ordering costs comprise $503.93 and holding costs $5,270.79. The ready rate is 95%, meeting the service level requirement. In short, there is no error in performance at all.

4.3 Comparison

In this section five LTD models have been presented: the full model LTD*, the reduced model LTD_0, and three adjusted models LTD_L, LTD_{LD}, and LTD_V. The table below summarizes the five models for the example item.

Table 8: Performance Characteristics of LTD*, LTD_0, LTD_L, LTD_{LD}, and LTD_V

	LTD*	LTD_0	LTD_L	LTD_{LD}	LTD_V
D	10	10	10	0.586	10
L	14	14	17.5	239	14
V	4	4	4	4	68.3
r	178.8	144.8	179.3	187.9	178.8
Q	36.2	24.4	24.8	13.8	36.2
***C* (realized)**	$5,774.72	$2,868.66	$5,552.93	$6,397.66	$5,774.72
***RR* (realized)**	95.0%	72.0%	93.9%	95.0%	95.0%
***C* (expected)**		$2,312.54	$2,261.14	$5,154.99	$5,774.72
***RR* (expected)**		95.0%	93.0%	95.0%	95.0%

The best model is the full model, LTD*, with parameters $(\mu_D, \sigma^2_D, \mu_L, \sigma^2_L)$, as it is the most accurate and relatively simple to solve. Implementing LTD*, however, can require significant effort to collect lead time data, calculate σ^2_L, and update the optimization models. The reduced model LTD_0, with parameters *(μ_D, σ^2_D, μ_L, 0)* ,is conceptually simple and relatively easy to implement, but as is clear in the above table, the results are quite inaccurate.

The mean inflation model, LTD_L, with parameters *(μ_D, σ^2_D, L, 0)*, has the advantage that it is fairly intuitive and more accurate than LTD_0. The disadvantages, however are that finding *L* is computationally intensive and harder to implement, requiring solving a bilevel nonlinear optimization problem as it does, and that LTD_L cannot match LTD* exactly. The joint mean inflation model LTD_{LD}, with parameters *(D, σ^2_D, L, 0)*, has the advantage of matching the moments of LTD* exactly, but it has the disadvantage that LTD_{LD} is somewhat counterintuitive and still does not have the same optimal policy or performance measures. The variance inflation model LTD_V, with parameters *(μ_D, V, μ_L 0)*, is not as intuitive as LTD_L, but it does match the moments and functional form of LTD* exactly and is mathematically speaking, the best model after LTD*. In addition, it is somewhat simpler to implement than LTD_L.

5 COMPUTATIONAL COMPARISON

A computational experiment was carried out to compare the performance of the full, reduced, and adjusted models. This section discusses the test cases, computational experiment, and analysis of the results.

5.1 TEST CASES

The models were tested over items classified as AAC-D at DLA, which have relatively high demand and low variability. Preliminary investigations were conducted on a test bed of 5733 SKUs, after which a refined dataset was developed. The dataset used for the final experiments was compiled from two files provided by DLA (Table 9). The first file, LT_ERROR.xls contained lead time moments for 15,725 items. The second file, ipo_celdi.txt, contained data for all 226,311 AAC-D SKUs handled by DLA's current inventory optimization software IPO. The SKUs consisted of item plus location. There were 156,888 distinct items and 88 distinct locations.

These two files were combined to create an input file for the computational experiments. The final dataset only retained those SKUs whose item ID was listed in LT_ERROR.xls. In the absence of further data, DLA confirmed that it was not inaccurate to assume the lead time per item was the same across all locations. The synthesized input file, dataset.csv, included 11,019 SKUs, corresponding to 6,943 distinct items across 66 distinct locations. Since the dataset is not a random sample of the SKUs, the test cases may not necessarily be representative of all AAC-D SKUs.

Table 9: Test Cases – Sources and Files

File	Source	Type	SKUs	Items	Locations	Parameters
LT_ERROR.xls	DLA	raw data	not included	15,725	not included	μ_L, σ^2_L
ipo_celdi.txt	DLA (IPO database)	raw data	226,311	156,888	88	μ_D, σ^2_D, K, i, c, γ
dataset.csv	synthesized by CELDi team	input to experiments	11,019	6,943	66	μ_D, σ^2_D, μ_L, σ^2_L, K, i, c, γ

The table below presents a summary of the characteristics of the dataset used in testing. The mean lead times contained in this dataset range from 3 weeks to 2 years. The cost per order K was close to \$40.66 for all items. It is our guess that other values represent rounding or data-entry error in the dataset. Likewise, the holding rate i for all items was 12% per year or 0.033% per day. Service levels ranged widely, but were generally fairly high – the median was 98.5%.

Table 10: Summary of Testing Dataset (Time units are days)

Parameter	μ_D	σ^2_D	μ_L	σ^2_L	K	i	c	γ
Mean	7.67	311,581.99	139.14	6,075.98	\$40.66	0.0329%	\$343.52	95.2%
Variance	9,364.04	1.75E+14	4,685.75	97,396,848	\$0.00	0.0000%	\$1,163,574.48	1.0%
Std. Dev	96.77	13,212,206.34	68.45	9,868.98	\$0.00	0.0000%	\$1,078.69	10.0%
Mode	0.38	0.0548	94	1,702	\$40.66	0.0329%	\$0.03	100.0%
Min	0.000063	0.000004	23	1	\$40.64	0.0329%	\$0.00378	1.5%
Q1	0.04553	0.2189	91	1,101	\$40.66	0.0329%	\$5.25	95.2%
Median	0.1715	2.265	125	2,787	\$40.66	0.0329%	\$34.86	98.5%
Q3	0.8745	55.19	173	7,003	\$40.66	0.0329%	\$220.22	99.6%
Max	5,966.50	1,258,956,079	670	163,770	\$40.87	0.0329%	\$21,396.33	100.0%

5.2 LTD MODELS

This section describes the LTD models considered and how they were compared. First some error measures are defined to compare the reduced models LTD_0 and LTD_L to LTD_* over the test cases . The performance of LTD_0, LTD_L and LTD_* were compared over the test cases described in the previous section. Since LTD_{LD}, is less accurate than LTD_V for the same amount of computational effort, it was dropped from further consideration. For each of LTD_0, LTD_L and LTD_*, the optimal policy parameters (r^*_i, Q^*_i) and adjusted input parameters (e.g. L^* or V) were found. The remainder of this section defines error measures, discusses model details, and summarizes the results.

5.2.1 Error Metrics

This difference between the reduced and full policies can be measured using any pair of the performance measures in section 4.1.2. Both the deviation (error) and relative error are important. Because cost and ready rate have different magnitudes, the mean squared relative error (MSRE) of relevant cost and ready rate is used as a summary statistic. Three performance measures (best, expected, realized) lead to defining the error three different ways – expected vs. best, realized vs. best, and expected vs. realized – each of which is presented below. The generic formulas for all the error measures are:

$$Error = Actual - Predicted$$

$$RE = \frac{Error}{Actual} = \frac{Actual - Predicted}{Actual}$$

$$MSRE = \frac{1}{2}\left[(RE(Cost))^2 + (RE(Ready\ Rate))^2\right]$$

5.2.1.1 Expected vs. Best

The first error measure compares optimal performance predicted by LTD_0 (i.e. planned performance) to that predicted by the full model LTD*. This error measure is mentioned in passing in Bagchi et. al. (1986), where it was called EVAI (expected value of additional information). Using this error measure might be appropriate in cases where planning decisions are made using the reduced model and the analyst was interested in tracking how far the model predictions were from the best possible performance. This speaks to questions of whether switching to the full model is worthwhile.

- Use: How far are reduced model predictions from best possible performance?
- Is switching to the full model worthwhile?

5.2.1.2 Expected vs. Realized

The error measure compares the performance of (r_0^*, Q_0^*) as predicted by LTD_0 and LTD_*. If the conditions of the full model actually apply but planning decisions are made using the reduced model, this error measure would capture the error between the predicted and operational performance. This speaks to the risk of prediction inaccuracy due to the assumption of constant lead time.

- Use: How far are reduced model predictions from realized performance?
- What does the reduced model cost you?

5.2.1.3 Realized vs. Best

The final error measure compares the performance of the optimal policies (r_0^*, Q_0^*) and (r^*, Q^*) as predicted by the full LTD model. If the conditions of the full model actually apply but planning decisions are made using the reduced model, this error measure would capture how far operational performance falls short of the best possible performance. This speaks to the risks of operational underperformance due to the assumption of constant lead time.

- Use: How far is the realized performance from the best possible performance?
- What is the inferior performance costing you?

5.2.2 Example

The error between LTD_0 and LTD_* for the example item is calculated below and summarized in Table 11. The error calculations using an adjusted reduced model such as LTD_L are similar.

5.2.2.1 Expected vs. Best

$$Cost\ Error = C(r^*, Q^*|LTD_*) - C(r_0^*, Q_0^*|LTD_0)$$
$$Cost\ Error = \$5{,}774.72 - \$2{,}312.54 = \$3{,}462.18$$

$$RE(Cost) = \frac{3{,}462.18}{5{,}774.72} = 60.0\%$$

$$Ready\ Rate\ Error = RR(r^*, Q^*|LTD_*) - RR(r_0^*, Q_0^*|LTD_0)$$
$$Ready\ Rate\ Error = 95.0\% - 95.0\% = 0$$

$$RE(Ready\ Rate) = \frac{0\%}{95\%} = 0\%$$

$$MSRE = \frac{1}{2}[0.600^2 + 0^2] = 0.1797$$

5.2.2.2 Expected vs. Realized

$$Cost\ Error = C(r_0^*, Q_0^*|LTD_*) - C(r_0^*, Q_0^*|LTD_0)$$
$$Cost\ Error = \$2{,}868.66 - \$2{,}312.54 = \$556.12$$

$$RE(Cost) = \frac{556.12}{2{,}868.66} = 19.4\%$$

$$Ready\ Rate\ Error = RR(r_0^*, Q_0^*|LTD_*) - RR(r_0^*, Q_0^*|LTD_0)$$
$$Ready\ Rate\ Error = 72.0\% - 95.0\% = -23.0\%$$

$$RE(Ready\ Rate) = \frac{-23.0\%}{72.0\%} = -32.0\%$$

$$MSRE = \frac{1}{2}[0.194^2 + (-0.320)^2] = 0.06981$$

5.2.2.3 Realized vs. Best

$$Cost\ Error = C(r^*, Q^*|LTD_*) - C(r_0^*, Q_0^*|LTD_0)$$
$$Cost\ Error = \$5{,}774.72 - \$2{,}868.66 = \$2{,}906.06$$

$$RE(Cost) = \frac{2{,}906.06}{5{,}774.72} = 50.3\%$$

$$RR\ Error = RR(r^*, Q^*|LTD_*) - RR(r_0^*, Q_0^*|LTD_*)$$
$$RR\ Error = 95.0\% - 72.0\% = 0$$

$$RE(RR) = \frac{72.0\%}{95.0\%} = 24.2\%$$

$$MSRE = \frac{1}{2}[0.503^2 + 0.242^2] = 0.1559$$

Table 11: Error Measures for Example Item

	Predicted	
Actual	**Expected** $(\boldsymbol{r_0^*}, \boldsymbol{Q_0^*}|\boldsymbol{LTD_0})$	**Realized** $(\boldsymbol{r_0^*}, \boldsymbol{Q_0^*}|\boldsymbol{LTD_*})$
Best $(\boldsymbol{r^*}, \boldsymbol{Q^*}|\boldsymbol{LTD_*})$	Cost: \$3,462.18 (60.0%) Ready Rate: 0% (0%) MSRE 0.3594	Cost: \$2,906.06 (50.3%) Ready Rate 23.0% (24.2%) MSRE: 0.2477
Realized $(\boldsymbol{r_0^*}, \boldsymbol{Q_0^*}|\boldsymbol{LTD_*})$	Cost: \$556.12 (19.4%) Ready Rate: -23.0% (-32.0%) MSRE: 0.06981	Key: Cost Error (RE) RR Error (RE) MSRE

5.2.3 Mean Inflation Model

In this model, the lead time is treated as a constant but its value L is inflated to account for lead time uncertainty. For each test case, an optimal inflated lead time L^* was found by minimizing the MSRE of best vs. realized performance with KPIs relevant cost and service level. L^* and policy parameters (r_L^*, Q_L^*) were found by solving the following bilevel nonlinear optimization problem.

$$\text{Min } MRSE = \frac{1}{2}\left(\left[\frac{C(r^*,Q^*|LTD_*) - C(r_L^*,Q_L^*|LTD_*)}{C(r^*,Q^*|LTD_*)}\right]^2 + \left[\frac{\gamma - RR(r_L^*,Q_L^*|LTD_*)}{\gamma}\right]^2\right)$$

S.t.

(r_L^*, Q_L^*)=*argmin{LC(r, Q, LTD*$_L$*)|* $\mu_L = L$ *and* $\sigma_L^2 = 0$ *}*

L >= 0

Other MSRE variants were explored in preliminary testing, including best vs. expected, realized vs. expected, and best vs. solution-to-solution. Best vs. expected MSRE and realized vs. expected MSRE tended to set L^* very, very high; best vs. solution-to-solution MSRE produced lower-quality solutions than best vs. realized MSRE.

The other issue explored was whether to include backorder cost in the MSRE calculation. Preliminary investigation found that when backorder costs were included, they tended to be the largest component of the cost error (since high service levels lead to backorder costs much larger than the holding costs). The solutions produced under this objective function tended to minimize the error in backorder costs at the expense of holding cost, ordering cost, and ready rate. Since backorder cost is only a Lagrangian multiplier term, not a KPI, it made more modeling sense to leave it out and use relevant cost in the MSRE calculation as opposed to Lagrangian cost.

5.3 Results

In this section the results of the computational experiments are described, concentrating on the distribution of service level error and cost percent error. As mentioned earlier, LTD_0, LTD_L, LTD_{CV}, and LTD_V are compared with LTD* over the 11,019-case test bed described earlier. It is very important to note that these results should be interpreted with caution, as the representativeness of the test bed cannot be evaluated. Note that the items in the test bed were not a random sample of the entire set of items.

Table 12, below, summarizes the results of the test cases. When LTV is taken into account, as in LTD* and LTD_V, the total annual cost is $13.4 million for the SKUs in the test data. The other three models, LTD_0, LTD_L, and LTD_{CV}, predict cheaper policies for these SKUs (annual costs of $9.5 million, $10.5 million, and $9.6 million) with the same service level. If implemented, however, LTD_0 and LTD_{CV} would result in an annual cost of $10.0 million but on average miss the service level target by 1.7%. Implementing LTD_L would result in total annual cost of $13.9 million, which is $400,000 more than LTD*, but closes 60% of the service level gap of LTD_0 and LTD_{CV}, cutting the average service level deficit to 0.684%. In the table the "Expected" rows correspond to the predicted values and the "Realized" rows to the implemented values. Since average service level deficit of the imperfect models LTD_0, LTD_L, and LTD_{CV} is not too great (0.68% to 1.74%), perhaps the cost savings are worth the service penalty. Of the imperfect policies, LTD_L gives the most accurate realized results. LTD_{CV}, on the other hand, performs very similarly to LTD_0 on realized cost and service level, implying that 30% of mean lead time is not the most accurate estimate of LTV. Table 18: Descriptive Statistics of Service Level and Cost Errors in the appendix contains further detail on the distribution of cost and service level errors.

Table 12: Summary of Results for Test SKUs. Negative error corresponds to an increase in cost or service level.

Model	Metric	Daily Cost (Holding and Ordering)	Annual Cost (Holding and Ordering)	Total Error (Daily)	Total Error (Annual)	Average Service Level Error
LTD*		$36,941.60	$13,483,700.00	$-	$-	0%
LTD_0	Expected	$26,105.00	$9,528,330.00	$10,836.60	$3,955,370.00	0%
LTD_0	Realized	$27,316.80	$9,970,630.00	$9,624.80	$3,513,070.00	1.74%
LTD_L	Expected	$28,701.90	$10,476,200.00	$8,239.70	$3,007,500.00	0%
LTD_L	Realized	$38,074.80	$13,897,300.00	-$1,133.20	-$413,600.00	0.684%
LTD_{CV}	Expected	$26,241.20	$9,578,040.00	$10,700.40	$3,905,660.00	0%
LTD_{CV}	Realized	$27,430.20	$10,012,000.00	$9,511.40	$3,471,700.00	1.70%
LTD_V		$36,941.60	$13,483,700.00	$-	$-	0%

Another way to evaluate the accuracy of a reduced model is to look at what fraction of the test cases fall within certain neighborhoods of the true value. For example, if LTD* predicts an optimal cost of $10,000/year with a service level of 98%, a reduced model whose optimal policy has a realized cost of $9,950/year and a service level of 97.5% might be considered "good enough for practical purposes". On the other hand, a reduced model whose optimal policy costs $11,000/year and has a service level of 90% is much less useful for decision-making.

This is formalized in a metric called the probability of error or PE, which is defined as:

$$PE(S, Metric) = \Pr\,(Error[Metric] \in S)$$

When S is specified separately, this is written *PE(Metric).* Since target service levels are already expressed in percentages, PE is a good way to summarize these errors. The formulas can be written:

$$PE\big((-\infty, -.1), Service\big) = \Pr\,(\gamma - Realized\ RR < -0.1)$$

$$PE([0.01, 0.05), Service) = \Pr\,(0.01 \leq \gamma - Realized\ RR < 0.05)$$

Using PE, we can therefore examine what fraction of the test bed has a realized service level within 1%, 5%, and 10% of the optimal cost of the best policy. Figure 2 shows *PE(Service Level)* for LTD_0, LTD_L, LTD_{CV}, and LTD_V. For LTD_V, all test cases fall within the 0-1% band (because error is 0% for all cases). For LTD_0 and LTD_{CV} 63% of the cases have a service level within 1% of the target and only 3.4% have a service level more than 10% away from the target. LTD_L, however, does even better, with 82% of the cases falling within 1% of the target and < 1% more than 10% away.

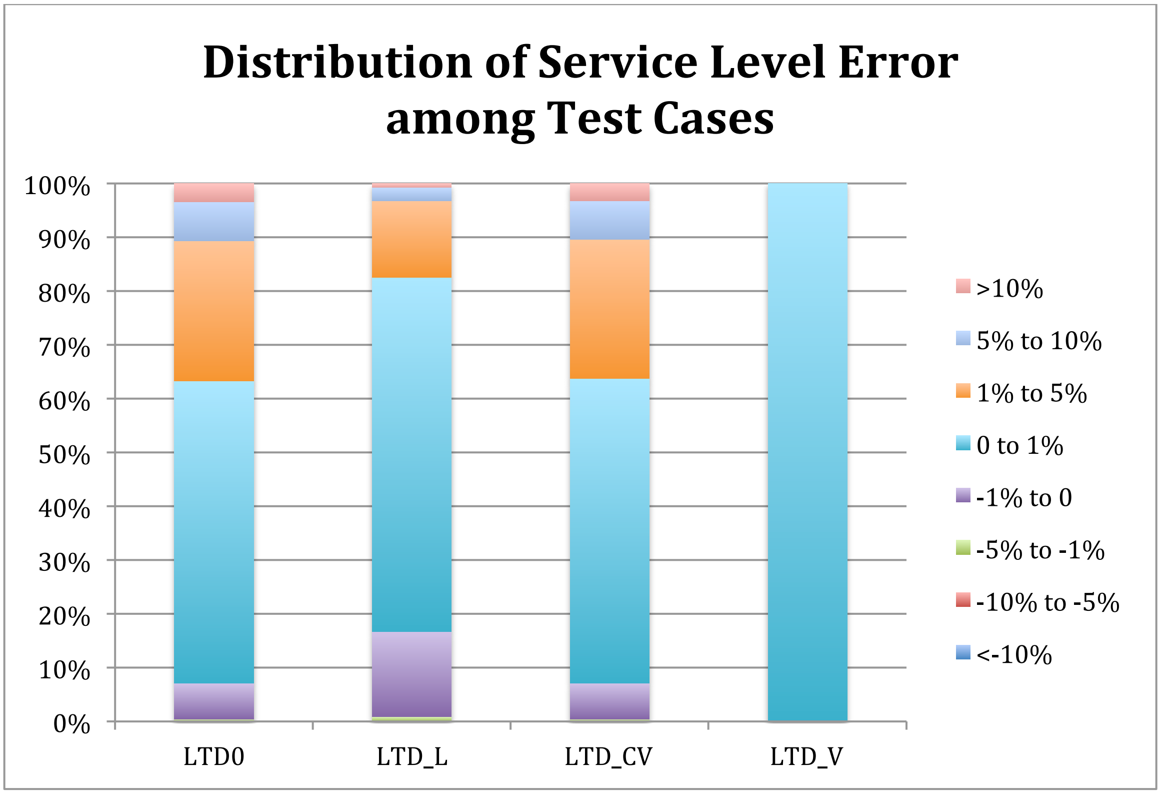

Figure 2: PE(Service) bins by LTD model

The probability of relative error (PRE) can be defined similarly to PE. Since realized cost varies so much across SKUs, it makes more sense to scale the cost errors using PRE. The formulas are:

$$PRE(S, Metric) = \Pr\left(RelError[Metric] \in S\right)$$

$$PRE\big((-\infty, -.1), Cost\big) = \Pr\left(\frac{Actual\ Cost - Realized\ Cost}{Actual\ Cost} < -0.1\right)$$

$$PRE([0.01, 0.05), Cost) = \Pr\left(0.01 \leq \frac{Actual\ Cost - Realized\ Cost}{Actual\ Cost} < 0.05\right)$$

Figure 3 shows the PRE(Cost) for the same LTD models and has a similar pattern, but the errors are much larger. For LTD_V, all test cases fall within the 0-1% band (because error is 0 for all cases). For LTD_0 and LTD_{CV} only 19% of the cases have a realized cost within 1% of the target and 40% of the cases have a realized cost more than 10% away from the optimal cost of LTD*. LTD_L, however, does somewhat better, with 35% of the cases falling within 1% of the target and only 18.4% more than 10% away. For all three LTD models, however, only a minority of cases are that close to the optimal value. When the bands are widened to

+- 5% more cases are included. For LTD_0 or LTD_{CV}, 59% of the SKUs fall within 5% of the optimal cost. LTD_L still performs better, with 81% of the SKUs in this band..

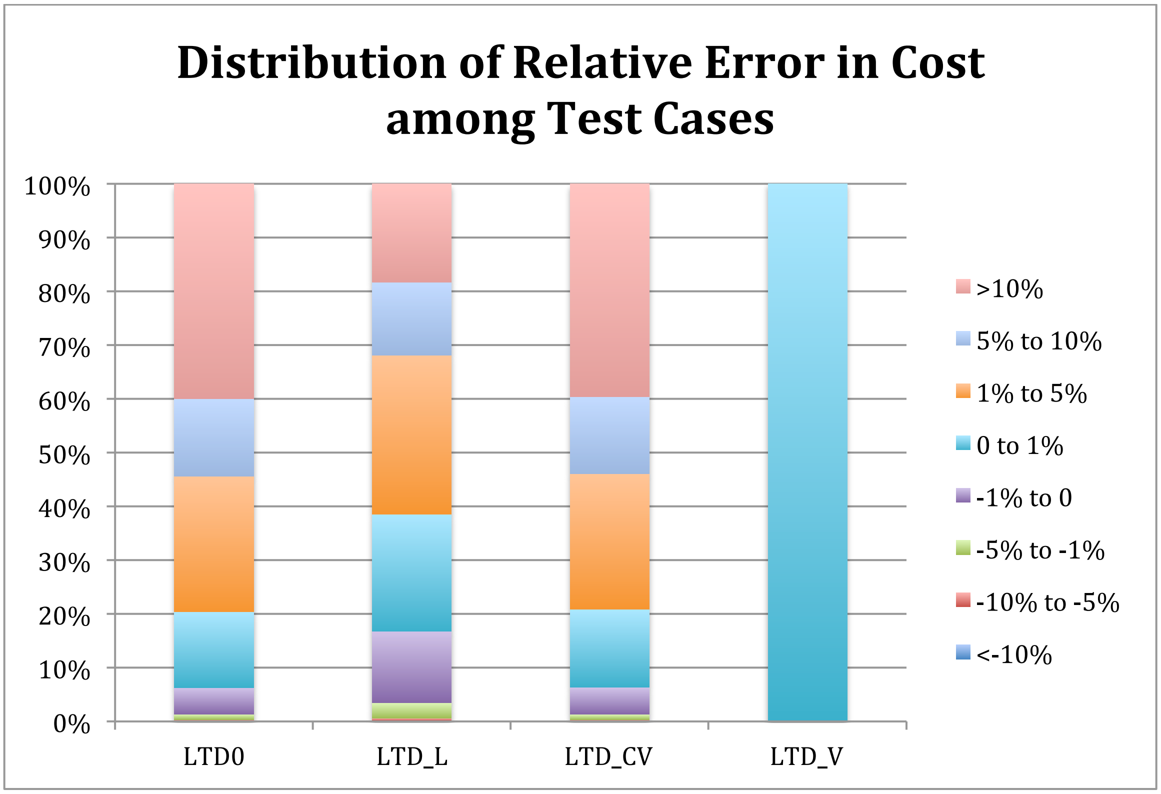

Figure 3: PRE(Cost) bin by LTD Model

In practice, however, service level and cost are inversely related, and their tradeoff is of interest. Table 13, Table 14, and Table 15 capture the joint distribution of PE(Service) and PRE(Cost) for LTD_0, LTD_L, and LTD_{CV}. In each table, the nonzero entries are concentrated near the diagonal, suggesting that PE(Service) and PRE(Cost) are correlated.

For this analysis, "accurate" corresponds to an error of +/- 1% in both service level and cost, and "semi-accurate" corresponds to a service level error of +/- 1% and a cost error of +/- 5%. LTD_V gives exactly the same results as LTD*, so 100% of the test cases fall into the "accurate" category. For LTD_0, (Table 13) 2,097 (19.0%) of the cases are accurate and 4,834 (43.9%) of the cases are semi-accurate. LTD_{CV} (Table 15) has similar numbers: 2,151 (19.5%) accurate cases and 4,890 (44.4%) semi-accurate. LTD_L (Table 14), however, does better, with 3,869 (35.1%) accurate at 7,313 (66.4%) semi-accurate.

Table 13: LTD_0 – Percent of Test Cases For Error Level Thresholds

Cost Thresholds	Service Level Thresholds <-10%	-10% to -5%	-5% to -1%	-1% to 0	0 to 1%	1% to 5%	5% to 10%	>10%	PRE(Cost)
<-10%	0%	0.0363%	0.1180%	0%	0%	0%	0%	0%	**0.154%**
-10% to -5%	0%	0%	0.1543%	0.0454%	0%	0%	0%	0%	**0.200%**
-5% to -1%	0%	0%	0.1361%	0.862%	0%	0%	0%	0%	**0.998%**
-1% to 0	0%	0%	0%	4.783%	0.163%	0%	0%	0%	**4.946%**
0 to 1%	0%	0%	0%	0.962%	13.12%	0.0091%	0%	0%	**14.09%**
1% to 5%	0%	0%	0%	0%	23.98%	1.243%	0%	0%	**25.22%**
5% to 10%	0%	0%	0%	0%	10.27%	4.138%	0.0182%	0%	**14.43%**
>10%	0%	0%	0%	0%	8.676%	20.65%	7.251%	3.385%	**39.96%**
PE(Service)	**0%**	**0.0363%**	**0.4084%**	**6.652%**	**56.21%**	**26.04%**	**7.269%**	**3.385%**	**100%**

Table 14: LTD_L – Percent of Test Cases For Error Level Thresholds

Cost Thresholds	Service Level Thresholds <-10%	-10% to -5%	-5% to -1%	-1% to 0	0 to 1%	1% to 5%	5% to 10%	>10%	PRE(Cost)
<-10%	0%	0.0363%	0.163%	0%	0%	0%	0%	0%	**0.200%**
-10% to -5%	0%	0%	0.354%	0.0545%	0%	0%	0%	0%	**0.408%**
-5% to -1%	0%	0%	0.345%	2.496%	0%	0%	0%	0%	**2.841%**
-1% to 0	0%	0%	0%	13.17%	0.145%	0%	0%	0%	**13.31%**
0 to 1%	0%	0%	0%	0.118%	21.68%	0%	0%	0%	**21.80%**
1% to 5%	0%	0%	0%	0%	28.76%	0.835%	0%	0%	**29.59%**
5% to 10%	0%	0%	0%	0%	9.974%	3.594%	0.0091%	0%	**13.58%**
>10%	0%	0%	0%	0%	5.209%	9.810%	2.478%	0.771%	**18.27%**
PE(Service)	**0%**	**0.0363%**	**0.862%**	**15.84%**	**65.77%**	**14.24%**	**2.487%**	**0.771%**	**100%**

Table 15: LTD_{CV} – Percent of Test Cases For Error Level Thresholds

Cost Thresholds	Service Level Thresholds -10% to -5%	-10% to -5%	-10% to -5%	-10% to -5%	-10% to -5%	-10% to -5%	-10% to -5%	-10% to -5%	Total
<-10%	0%	0.0363%	0.118%	0%	0%	0%	0%	0%	**0.154%**
-10% to -5%	0%	0%	0.154%	0.045%	0%	0%	0%	0%	**0.200%**
-5% to -1%	0%	0%	0.136%	0.844%	0%	0%	0%	0%	**0.980%**
-1% to 0	0%	0%	0%	4.855%	0.172%	0%	0%	0%	**5.028%**
0 to 1%	0%	0%	0%	0.962%	13.53%	0%	0%	0%	**14.49%**
1% to 5%	0%	0%	0%	0%	24.01%	1.189%	0%	0%	**25.20%**
5% to 10%	0%	0%	0%	0%	10.18%	4.138%	0.0182%	0%	**14.34%**
>10%	0%	0%	0%	0%	8.703%	20.48%	7.160%	3.258%	**39.60%**
PE(Service)	**0%**	**0.0363%**	**0.408%**	**6.707%**	**56.60%**	**25.81%**	**7.179%**	**3.258%**	**100%**

5.4 ABC ANALYSIS

An ABC classification analysis was performed in order to examine whether error was concentrated among higher cost items (classes A and B). Ideally, this analysis would suggest which items it is the most important to collect lead time data for, in order to reduce error. The classes being used to define item cost are listed below:

- A: 993 (9.0%) SKUs: 70% of total item cost
- B: 2479 (22.5%) SKUS: 25% of total item cost
- C: 7547 (68.5%) SKUs: 5% of total item cost

Using this approach, comparisons of the errors in realized cost and realized service level (ready rate) of LTD_0 vs. LTD* were performed. The results are depicted in the boxplots in Figure 4 and Figure 5. Figure 4 shows that the service level error mean and variance decreases as item cost decreases, but Figure 5 shows that the cost error does not. The service level only decreases slightly, so it has less operational impact. This raises the importance of percent error in cost, which is insensitive to item class. Therefore, ABC analysis does not seem to be an effective method of identifying items most sensitive to LTV underestimation, and is not a very effective analysis methodology in this situation.

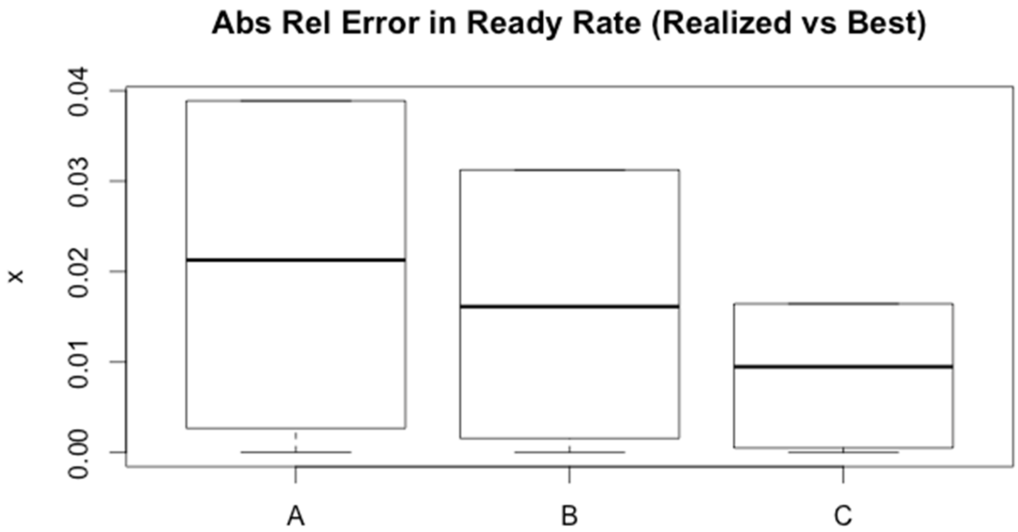

Figure 4: Percent error in service level (ready rate) by item class

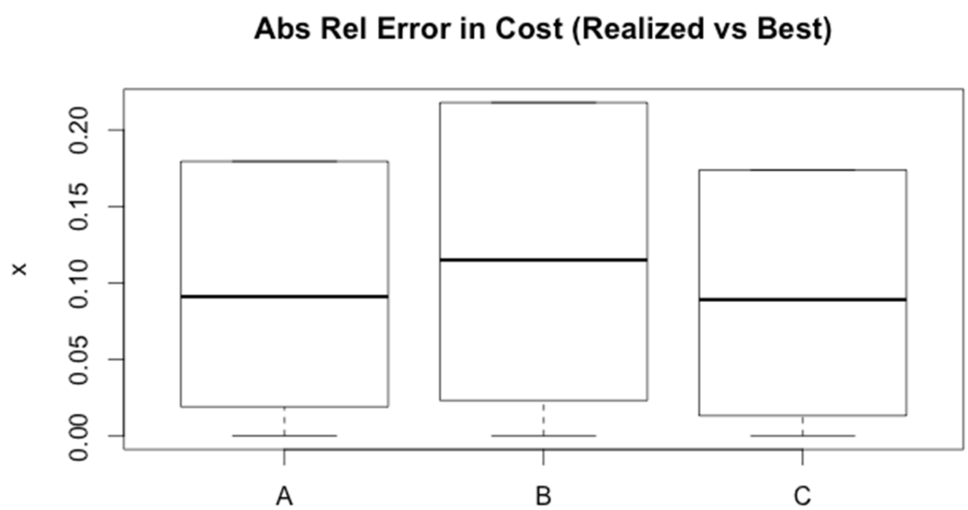

Figure 5: Percent error in relevant cost by item class

5.5 Predicting LTV by Regression

The feasibility of LTD_{CV} was investigated by fitting a linear regression model to the test dataset to predict LTV from the other input parameters. Since lead time information was provided for NIINs rather than SKUs, the SKU data was aggregated by location. Demand mean and variance were summed by location and service level was averaged across locations. The resulting dataset contained 6,943 unique NIINs. The predictors were $(\mu_D, \sigma_D^2, \mu_L, c, \gamma)$. The ordering cost K and the holding rate i were the same for all SKUs in the dataset, so they were dropped from the regression model.

A full second-order model was fit to the entire dataset and refined by dropping statistically insignificant interactions. In the fitted model, service level had significant interactions but was not itself significant, suggesting the presence of significant colinearity and casting doubt on the validity of the model.

The dataset was then broken up by item class (A, B, or C) and second-order regression models were fit. After removing insignificant terms from the full second order model, only the intercept and μ_L terms remained in the regression equation for A and B items. While some other terms remained in for C items, the R^2 for a regression equation with only μ_L as a predictor was very close (.5075 vs .4987). In other words, this partially validates the LTD_{CV} approach because the models with the highest or nearly highest R^2 only contained μ_L as a predictor. The R^2 for these models was very low (0.2514 for A items, 0.3708 for B, and 0.4987 for C) and the residuals of each model violated the assumptions of normally-distributed error and constant error variance. It is therefore not recommended to actually use these models; instead, in view of the low R^2 for the regression, it probably makes more sense to actually start recording and using LTV.

6 SOFTWARE PROTOTYPE

The mean inflation model, LTD_L, and the variance inflation model LTD_V are implemented in a Java program. This program takes a .csv file of SKU data and outputs the optimal input parameter settings (L* for LTD_L or V for LTD_V), policy settings, performance metrics, and errors as .csv files. This section is a user guide to the program.

6.1 IMPLEMENTATION

The models were implemented in Java using open source code. The software is licensed under the GPL. It is based off of the Java Simulation Library, developed by Dr. Manuel Rossetti and (r, Q) inventory optimization software developed for a prior CELDi project (UA11-INV). It also uses OpenCSV 2.3 for file input and Apache Commons Math for a nonlinear optimization solver.

When invoked, the program will solve either the mean inflation model (MIM or LTD_L) or the variance inflation model (VIM or LTD_V) for all the cases provided in an input file, finding the full and reduced optimal policy parameters and calculating the best, expected, and realized performance. MIM uses LTD_L as its reduced model (boxes 1, 2, and 3 in Figure 6); VIM uses LTD_0 (boxes 1, 4, and 5 in Figure 6).

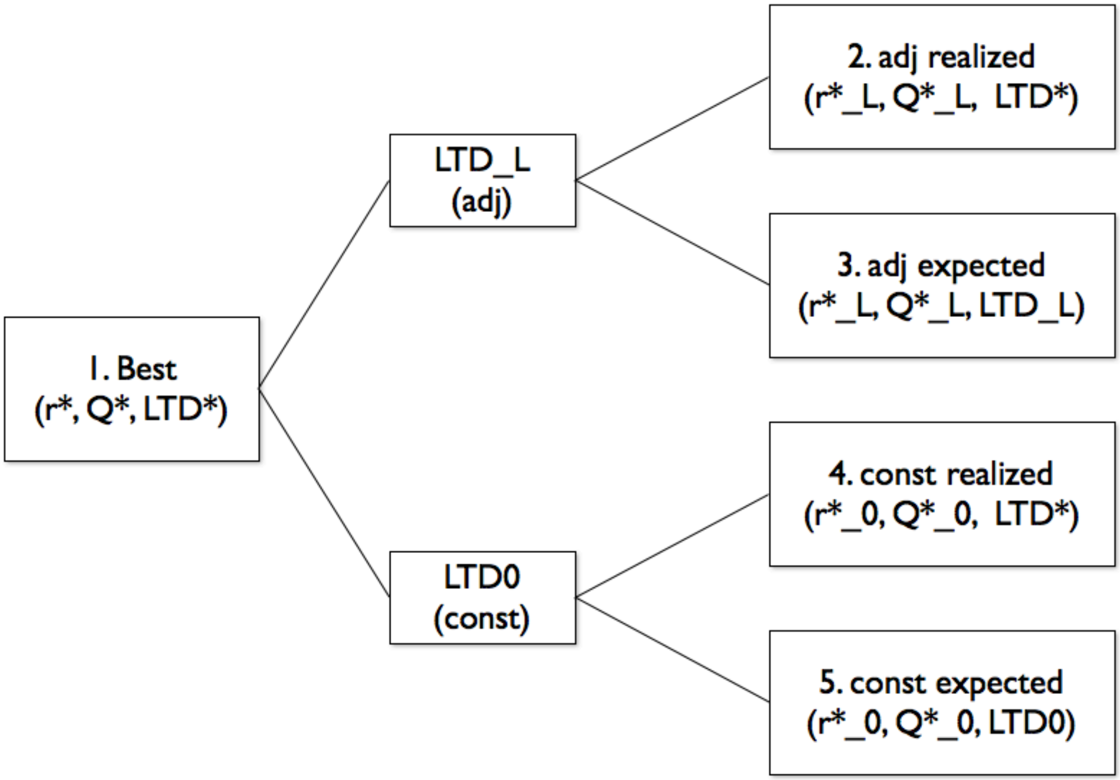

Figure 6: Performance measures for MIM

6.2 Input Format

The input file format is shown in Figure 7. The item data should be supplied as a .csv file. The program does not convert units, so it is important that time units, in particular, match. The holding rate *i* should be checked, as it is often expressed in $/$/year, while demand and lead time are more frequently given per day.

id	muD	varD	muL	varL	K	i	c	service
case0	0.1114	0.0603	169	5,711.1	21.89	0.000328767	462.98	0.5
case1	1.1400	4.2363	64.35	727.8	1.29	0.000328767	157.44	0.5
case2	0.0984	0.1503	222.83	36,166.3	12.37	0.000328767	1,883.66	0.5
case3	0.0932	0.0824	180.89	16,778.6	13.49	0.000328767	1,719.06	0.5

Figure 7: Input File Format

The program treats the first line of the file as a header and discards it. From left to right, the fields are:

1. Id: a label for the specific SKU. It is treated as text, even if expressed as a number.

2. muD: mean demand in units per time

3. varD: variance of demand
4. muL: lead time mean. The program assumes that the time units of muL are the same as muD.
5. varL: variance of lead time
6. K: the cost per order, in $/order
7. i: the holding rate, a percentage. The example uses i = .012/365 $/$/day
8. c: the unit cost, in $/item
9. svc: the target customer service level as a number from 0-1.

6.3 Invoking the program

The software must be run from the command line. The command to solve the variance inflation model LTD_V for the items contained in *filename.csv* is:

```
java –jar /path/to/jar/file/DLA.jar VIM filename.csv
```

The command to solve the mean inflation model LTD_L for the items contained in *filename.csv* is:

```
java –jar /path/to/jar/file/DLA.jar MIM filename.csv
```

Both the VIM and MIM models also optimize the full model and save the true optimal policy and performance measures in a file called *filename-best.csv* (see next section for explanation of output files). When only the full model is of interest, the VIM should be used because it runs more quickly than MIM.

6.4 Output Format

The program generates a number of output files in .csv format and places them in a folder named after the input file. The results for the input file *example.csv*, for example, will be stored in a folder called *example*. If the same file is run twice (e.g. to compare MIM and VIM), the older files will be overwritten. CSV format was chosen so that the output files may be further processed by the user.

6.4.1 List of Output Files

This section lists all the files generated by the program. These files reside in a directory with the same name as the input file. In the list below, the input file is called *example.csv*.

- *example_solution.csv*: This file lists the optimal policy for the full and reduced models and the appropriate inflated input parameter. For VIM, the inflated parameter is σ_D^2

and the reduced model is LTD$_0$ and the For MIM, the inflated parameter is μ_L and the reduced model is LTD$_L$. The fields are summarized in Table 16.

Table 16: Fields in example_solution.csv

Field Name	Models	Meaning
id	VIM, MIM	case identifier, usually SKU or NIIN
r*	VIM, MIM	Optimal reorder point for LTD_*
Q*	VIM, MIM	Optimal reorder size for LTD_*
IDV	VIM	Inflated Demand Variance V to replace σ_D^2 in LTD_V
ICL	MIM	Inflated Constant Leadtime L^* to replace μ_L in LTD_L
r*0	VIM	Optimal reorder point for LTD_0
Q*0	VIM	Optimal reorder size for LTD_0
r*L	MIM	Optimal reorder point for LTD_L
Q*L	MIM	Optimal reorder size for LTD_L

- Performance of optimal policies: Section 4.1.2 lists the three ways of measuring the performance of a reduced model: best, expected, and realized. Each one has a dedicated output file recording the optimal policy and its performance on many metrics. The most important ones are listed in Table 2, and a full list in Appendix A.
  - *example_best.csv* contains the full optimal policy (r^*, Q^*) and calculates performance using LTD_*.
  - *example_expected.csv*: For VIM, this file contains the reduced optimal policy (r_0^*, Q_0^*) and calculates performance using LTD_0. For MIM, this file contains the reduced optimal policy (r_L^*, Q_L^*) and calculates performance using LTD_L.
  - *example_realized.csv*: For VIM, this file contains the reduced optimal policy (r_0^*, Q_0^*) and calculates performance using LTD_*. For MIM, this file contains the reduced optimal policy (r_L^*, Q_L^*) and calculates performance using LTD_*.
- Error of reduced model: Section 5.2.1 list three ways of measuring the error of a reduced model, each of which is recorded in a dedicated output file. The error is calculated for each performance measure in Appendix A as (Actual – Predicted).
  - *example_error (B v E).csv*: Actual values are taken from *example_best.csv*; predicted values from *example_expected.csv*.
  - *example_error (B v R).csv*: Actual values are taken from *example_best.csv*; predicted values from *example_realized.csv*.

- *example_error (E v R).csv*: Actual values are taken from *example_realized.csv*; predicted values from *example_expected.csv*.

- Relative error of reduced model: These files record the relative or percent error corresponding to each of the files in the previous bullet. For each performance measure in Appendix A, the relative error is calculated as (Actual – Predicted)/Actual.
  - *example_pct (B v E).csv*: Actual values are taken from *example_best.csv*; predicted values from *example_expected.csv*.
  - *example_pct (B v R).csv*: Actual values are taken from *example_best.csv*; predicted values from *example_realized.csv*.
  - *example_pct (E v R).csv*: Actual values are taken from *example_realized.csv*; predicted values from *example_expected.csv*.

7 CONCLUSION

This report compared several LTD models for use in an *(r, Q)* inventory policy. Table 17 compares all five LTD models considered in this report: LTD*, LTD_V, LTD_0, LTD_{CV}, and LTD_L. LTD* is the best-practice model for *(r, Q)* inventory policies. The variance inflation model (LTD_V or VIM) always gives the same results as LTD* (total annual cost $13.5 million and no service level deficits) and is simple to implement.

Further research and modeling is needed to determine the best way to model items without historical data, such as new items. LTD_{CV}, estimating lead time variance as 30% of the mean, had a total annual cost of $10.0 million dollars and average service level deficit of 1.70%. These numbers do not represent a substantially improvement over LTD_0, the model where lead time variance was ignored completely, which had a total annual cost of $9.97 million and an average service level deficit of 1.74%. Linear regression was attempted over the test dataset, but the fitted models violated the assumptions of regression and had low R^2 values as well (ranging from 0.2514 to 0.5075). Therefore, simple linear regression is not recommended as an approach for this problem.

While LTD_L, the final LTD model considered, was an improvement over the current model LTD_{CV}, with total annual cost of $13.9 million and a service level deficit of only 0.684%, it requires the same historical data as LTD* or LTD_V but is more computationally intensive to use and less accurate. If historical data is available but the inventory system assumes lead time is constant, LTD_V is simpler to implement and more accurate than LTD_L and is therefore the recommended alternative. If the inventory system allows for stochastic lead time, then the lead time variance should be collected from historical data if possible and LTD* used.

Table 17: Summary of LTD Models

Model	LTD Parameters	Realized Annual Cost	Percent change in total cost over LTD* (negative if lower)	Average % decrease in service level	Percent of cases within 1% of LTD* for Cost and Service	Percent of cases with service level within 1% and cost within 5% of LTD
LTD*	$(\mu_D, \sigma_D^2, \mu_L, \sigma_L^2)$	$13,483,700	-	-		
LTD_V	$(\mu_D, V, \mu_L, 0)$	$13,483,700	0%	0%	100%	100%
LTD_0	$(\mu_D, \sigma_D^2, \mu_L, 0)$	$9,970,630	-26.1%%	1.74%	19.0%	43.9%
LTD_{CV}	$(\mu_D, \sigma_D^2, \mu_L, 0.3\mu_L)$	$10,012,000	-25.4%%	1.70%	19.5%	44.4%
LTD_L	$(\mu_D, \sigma_D^2, L, 0)$	$13,897,300	3.07%	0.684%	35.1%	66.4%

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9 APPENDIX A: DETAILED STATISTICS ON REDUCED MODEL ERRORS

Table 18: Descriptive Statistics of Service Level and Cost Errors, using the Realized vs. Best Error Metric

	Model	Mean	Var	Std Dev	Min	First Quart.	Median	Last Quart.	Max
Service Level Error	LTD_0	1.736%	0.0951%	3.084%	-9.315%	0.0487%	0.439%	2.018%	30.85%
	LTD_L	0.684%	0.0317%	1.779%	-9.713%	0.0054%	0.0953%	0.564%	20.67%
	LTD_{CV}	1.700%	0.0911%	3.019%	-9.296%	0.0465%	0.423%	1.973%	25.38%
	LTD_V	0%	0%	0%	0%	0%	0%	0%	0%
Annual Cost Error	LTD_0	$318.82	4,369,792	$2,090.40	-$2,661.99	$2.44	$23.39	$140.22	$154,898.16
	LTD_L	$203.49	3,307,946	$1,818.78	-$2,936.21	$0.32	$7.04	$56.63	$143,259.73
	LTD_{CV}	$315.06	4,254,027	$2,062.53	-$2,656.20	$2.35	$22.76	$138.28	$152,447.92
	LTD_V	$0	$0	$0	$0	$0	$0	$0	$0
Percent Error Service Level	LTD_0	1.799%	0.105%	3.247%	-24.110%	0.0497%	0.451%	2.116%	31.98%
	LTD_L	0.703%	0.0385%	1.962%	-26.198%	0.0055%	0.0969%	0.587%	21.00%
	LTD_{CV}	1.762%	0.101%	3.181%	-23.979%	0.0474%	0.437%	2.075%	26.31%
	LTD_V	0%	0%	0%	0%	0%	0%	0%	0%
Percent Error Cost	LTD_0	12.535%	2.360%	15.362%	-35.725%	1.455%	6.274%	18.27%	87.78%
	LTD_L	5.813%	1.013%	10.067%	-39.297%	0.305%	2.001%	6.864%	78.02%
	LTD_{CV}	12.384%	2.329%	15.261%	-35.631%	1.394%	6.121%	18.00%	87.59%
	LTD_V	0%	0%	0%	0%	0%	0%	0%	0%

Negative errors correspond to increases in realized cost or service level over LTD*.

10 APPENDIX B: USER GUIDE TO "LTD MODELS.XLSX"

In addition to the Java prototype, several of the models discussed in this report (LTD*, LTD_0, LTD_{CV}, LTD_L, and LTD_V) are also implemented in an Excel spreadsheet. The purpose of this workbook is to illustrate the impact of lead time variability on the performance of an *(r, Q)* inventory system on an item by item basis. This impact is chiefly meditated by the distribution of demand during replenishment lead time (LTD model).

10.1 WORKBOOK STRUCTURE

The workbook contains nine tabs. Problem parameters are entered in the first tab, "Input Parameters". The second and penultimate tabs, "Comparison" and "Summary", summarize the results and calculations. The middle tabs ("Full LTD Model", "LTD0", "LTD_L", "LTD_CV", and "LTD_V") calculate performance and optimal policy for a single LTD model. The final tab, "Lookups", contains various implementation details, none of which are relevant to the end-user.

10.2 THE "INPUT PARAMETERS" TAB

An *(r, Q)* stochastic inventory model has two main types of parameters: cost information and random variable moments. Throughout this guide, the spreadsheet functionality will be illustrated with an example. Table 19 duplicates Table 3 and summarizes the necessary parameters and gives the data for the example item.

Table 19: Inputs to (r, Q) model and example data

Time units	periods	Periods or time buckets	days
Cost/service	K	Ordering cost	$5
	c	Item cost	$10
	i	Holding charge per item per period	0.05
	γ	Target fill rate (customer service)	0.95
Random variables	μ_D	Mean demand per period	10
	σ_D	Standard deviation of demand per period	2
	μ_L	Mean lead time in periods	14
	σ_L	Standard deviation of of lead time in periods	3

This information is entered into the "Input Parameters" tab as shown in Figure 8 below. Data input cells have been shaded. For implementation purposes, $h=ic$, the period holding cost per item, and $b = \frac{\gamma h}{1-\gamma}$, the period backorder cost per item, are calculated. Setting b using this formula ensures that optimal policies will meet the target service level. If desired, holding cost h and backorder cost b may be entered directly in the relevant cells, but this will overwrite the formula that does the automatic calculations.

Time units must also be provided for holding charge, demand, and lead time. If demand and lead time are given in different units (e.g. days and weeks), the analysis will report the results using the shorter (e.g. days). Holding costs and backorder costs will also be converted to the appropriate time units. For example, if demand is given in items/day, the holding charge may be entered as 15% per item per year and but a daily holding charge will be calculated.

H15

	A	B	C	D	E	F
1	**Inputs**			**True LTD Distribution**		
2	**Costs**			Time units	day	
3	Item cost ($/item)	$ 10.00		Mean	140	items
4	Holding Charge ($/$/time)	0.25	per day	Var	956	
5	Holding Cost ($/item/time)	$ 2.50	per item per day	Std dev	30.919	items
6	Target Fill Rate	0.95		Alpha	20.5	
7	Backorder cost ($/item/time)	$ 47.50	per item per day	Beta	0.14644351	
8	Ordering Cost ($/order)	$ 5.00				
9				**Problem Characteristics**		
10				EOQ	6.325	items
11	**Demand Distribution**	Time unit		Qlb	6.489	items
12	Time unit	day		Critical Ratio	0.950	
13	Mean	10				
14	Std dev	2		**True Values**	**Expected Val**	**Units**
15				Demand epochs	365	days per year
16	**Lead Time Distribution**	Time unit		Lead time epochs	365	days per year
17	Time unit	day				
18	Mean	14				
19	Std dev	3				
20						
21						
22						
23						

Item Parameters | Comparison | Full LTD Model | LTD0 | LTD_CV | LTD_L | LTD_V | Summary | Lookups

Figure 8: Input Parameters tab, with data and results for the example item

The left side of the "Input Parameters" sheet lists and calculates some problem characteristics: the moments of the full LTD model (LTD*), the parameters of a Gamma distribution fitted to LTD*, EOQ, a sharper lower bound Q_{lb}, the critical ratio, and conversion factors for time units. See Section 3: The Standard (r, Q) Inventory Model (page 3) for formulae and a more extensive discussion.

10.3 The "Full LTD Model" tab

This tab implements LTD*, the standard academic (r, Q) inventory control model. What distinguishes the model in this tab from the models in the following tabs is that LTD* uses the full LTD distribution discussed in the previous section, explicitly incorporating lead time variance.

Figure 9 shows the spreadsheet implementation of LTD$_*$, in the "Full LTD Model" tab. The left half of the spreadsheet shows the probability distributions used in this model. At top is the demand distribution. In the middle is the lead time distribution. At the bottom is the LTD distribution. All are assumed to have a Gamma distribution. The parameters of each Gamma distribution are calculated beneath the mean and standard deviation. The moments are copied from the "Input Parameters" sheet. This half of the spreadsheet is for information and does not require any input.

On the right is policy information. The performance for any *r* and *Q* may be found by entering the appropriate values in the two cells labeled "r*" and "Q*". The "Constraint" section, below that, is used in the optimization model described in Section 3.2. Next, non-cost performance measures are listed. The final section gives long-run average cost performance measures. See Table 2 for a list of performance measures and their meanings.

Demand Distribution			**Policy Parameters**			**Solver Model**
Time unit	day		r*	186.685	items	$ 365.44
Mean	10	Items per day	Q*	16.546	items	2
Std dev	2	items				TRUE
Alpha	25	Items per day	**Constraint**			TRUE
Beta	2.5		r >= -Q	-16.546		32767
						0
			Performance Measures			
Lead Time Distribution			On Hand	55.80	items	
Time unit	day		Backorders	0.846	items	
Mean	14	days per delivery	Ready Rate	95%		
Std dev	3	days	Order Frequency	0.604	Orders per day	
Alpha	21.7777778	days per delivery	Safety Stock	46.685	items	
Beta	1.55555556					
			Expected Costs	**$ per day**	**$ per year**	
True LTD Distribution			Total Cost	$ 365.44	$ 133,387.24	
Mean	140	items	Ordering Cost	$ 6.04	$ 2,205.99	
Var	956		Holding Cost	$ 279.02	$ 101,842.81	
Std dev	30.9192497	items	Backorder Cost	$ 80.38	$ 29,338.44	
Alpha	20.5020921		Value of On Hand Inventory	$ 5,580.43	$ -	
Beta	0.14644351		Safety Stock Value	$ 4,668.53	$ -	
			Operating Cost	$ 285.07	$ 104,048.81	

Figure 9: Screenshot of the "Full LTD Model" tab, showing the optimal policy for the example item and its performance.

To find the optimal *(r, Q)* policy, use Solver, which is in the Data tab. If the Solver add-in is not visible in the Data tab, it will be necessary to install it. Installation is generally simple – a matter of checking a box in the right dialog box. The exact location can vary from version to version and operating system to operating system; consult Excel's help for exact instructions on your system.

The optimization model has already been created in this spreadsheet, so the procedure to find *(r*, Q*)* is fairly straightforward:

1. Click on Solver (Data tab)
2. In the Solver dialog box (Figure 10), hit "Solve".

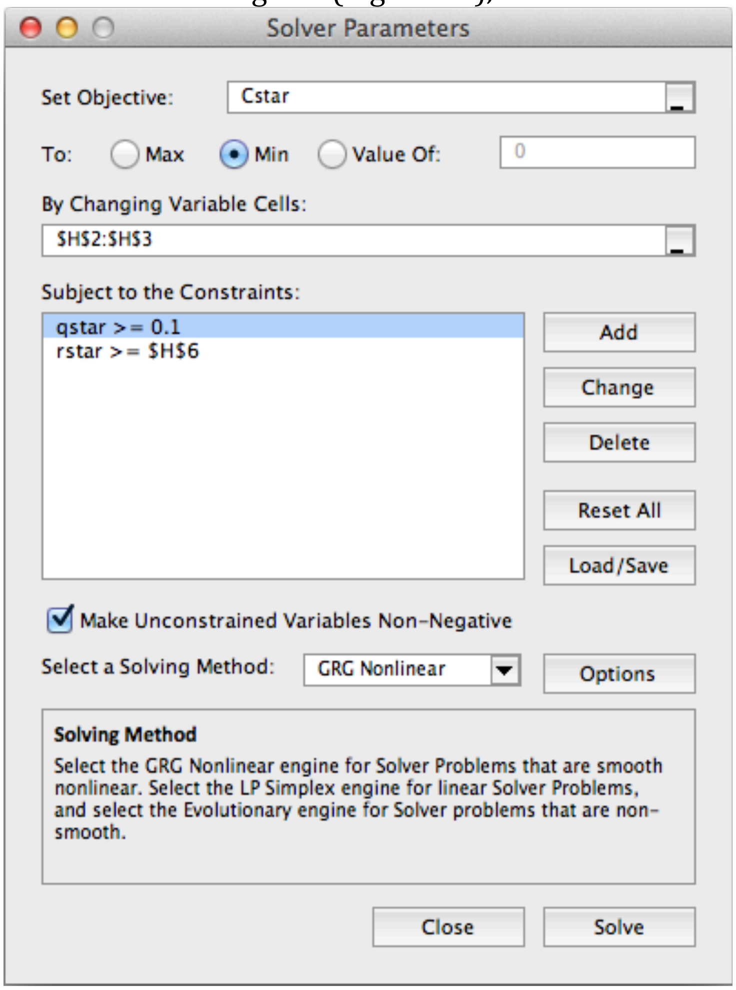

Figure 10: Solver dialog box with correct optimization model.

3. When the Solver Results dialog box pops up (Figure 11), hit OK.

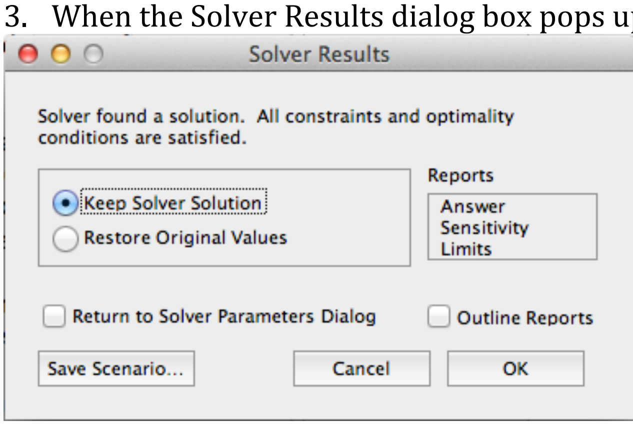

Figure 11: Solver Results dialog box.

4. Figure 9 shows the optimal policy for the example item.

10.4 THE “LTD0” TAB

This tab implements LTD_0, the *(r, Q)* inventory model that assumes constant or deterministic lead time (Section 4, page 9). In effect, lead time variance is ignored. In effect, this model is what happens when $(\mu_D, \sigma_D^2, \mu_L, \sigma_L^2)$ used for LTD_* is replaced with $(\mu_D, \sigma_D^2, \mu_L, 0)$. Since LTD_0 is not equivalent to LTD*, this tab also includes all three performance metrics discussed in Section 4.1.2: Expected vs. Best, Realized vs. Best, and Expected vs. Realized.

10.4.1 USAGE

A	B	C	D	E	F	G	H	I	J	K	L	M	N
Demand Distribution						**Policy Parameters**			**Solver Model**				
Time unit	day					r'	148.4	items	$ 47.29				
						Q'	10.08	items	2				
Mean	10	Items per day							TRUE				
Std dev	2					**Results**	**MSRE**		TRUE				
Alpha	25					Expected vs Best	27.47%		32767				
Beta	2.500					Realized vs Best	71.13%		0				
						Expected vs Realiz	45.97%						
Lead Time Distribution						**Constraints**							
Mean	14.00	days per delivery				r >= -Q	-10.08						
Std dev	0												
						Performance	Expected	SRE (Expe	Realized	SRE (Realizec	SRE (Expecte	Units	
LTD Distribution						On Hand	13.58	57.26%	20.53	39.96%	11.46%	items	
Mean	140.0	items				Backorders	0.18	62.64%	7.13	5509.40%	95.11%	items	
Var	56.00					Ready Rate	95%	0.00%	69%	7.59%	14.46%		
Std dev	7.483	items				Order Frequency	0.992	41.11%	0.992	41.11%	0.00%	Orders per day	
Alpha	350.0					Safety Stock	8.36	67.39%	8.360	67.39%	0.00%	items	
Beta	2.5												
						Expected Costs	Expected	SRE (Expe	Realized	SRE (Realizec	SRE (Expecte	Units	
						Total Cost	$ 47.29	54.94%	$ 394.77	134.67%	77.48%	Dollars per day	
						Ordering Cost	$ 4.96	41.11%	$ 4.96	41.11%	0.00%	Dollars per day	
						Holding Cost	$ 33.94	57.26%	$ 51.32	39.96%	11.46%	Dollars per day	
						Backorder Cost	$ 8.38	62.64%	$ 338.49	5509.40%	95.11%	Dollars per day	
						Value of On Hand Inventory	$ 135.77	57.26%	$ 205.27	39.96%	11.46%	Dollars	
						Safety Stock Value	$ 83.60	67.39%	$ 83.60	67.39%	0.00%	Dollars	
						Annual Relevant Cost	$ 14,199.55	52.86%	$ 20,541.13	36.62%	9.53%	Dollars per year	

Figure 12: Screenshot of the “LTD0” tab, showing the optimal policy under LTD_0 for the example item and its performance.

Figure 12 shows the spreadsheet implementation of LTD_0. The setup is similar to “Full LTD Model” tab (Figure 9). The left side of the worksheet is very similar. The demand moments at the top have not changed. In the middle, the lead time mean is the same; only the “Std Dev” has changed and is now 0. The LTD distribution is automatically calculated in the bottom section.

On the right side of the, the performance for any *r* and *Q* may be found by entering the appropriate values in the two cells labeled “r*” and “Q*”. The next section calculates the

MSRE of LTD_0 according to each of the variants listed above: Expected vs. Best, Realized vs. Best, and Expected vs. Realized. The “Constraint” section, below that, is used in the optimization model.

The performance measure sections now contain much more information. The first column, “Expected”, calculates the performance of r and Q given at the top of the right side of the sheet, calculated used the LTD model at the bottom of the left side of the worksheet. The next column, “SRE (Expected)”, gives the squared relative error (terms in MRSE) between the values in the previous column and those in the “Full LTD Model” sheet, implementing the “Expected vs. Best” MSRE; (for LTD* values see Figure 9)

The third column, “Realized”, calculates the performance of the r and Q given in this sheet using LTD_* -- that is, the parameters in “Full LTD Model”, left side, bottom section. The next column, “SRE (Realized vs. Best)” contains the error between the “Realized” column and the values in “Full LTD Model”, implementing the “Realized vs. Best” MSRE (for LTD* values see Figure 9). The penultimate column is the error between the “Expected” and “Realized” columns on this sheet and implements the “Expected vs Realized” MSRE. The final column contains units.

The optimization model has already been created in this spreadsheet, so the procedure to find (r_0^*, Q_0^*) is fairly straightforward and similar to “Full LTD Model”.

1. Click on Solver (Data tab)
2. In the Solver dialog box (Figure 13), hit “Solve”.

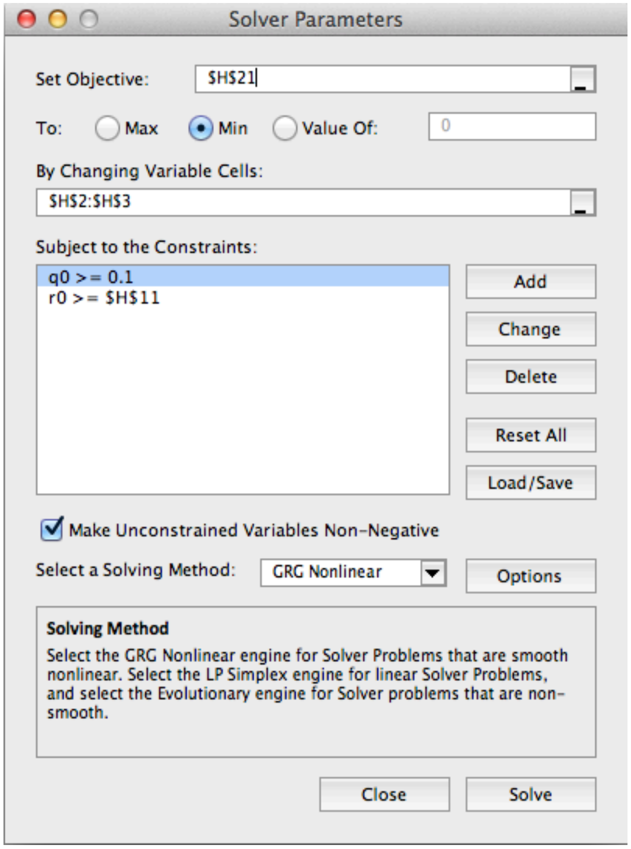

Figure 13: Solver dialog box with correct optimization model.

3. When the Solver Results dialog box pops up (Figure 14) hit OK.

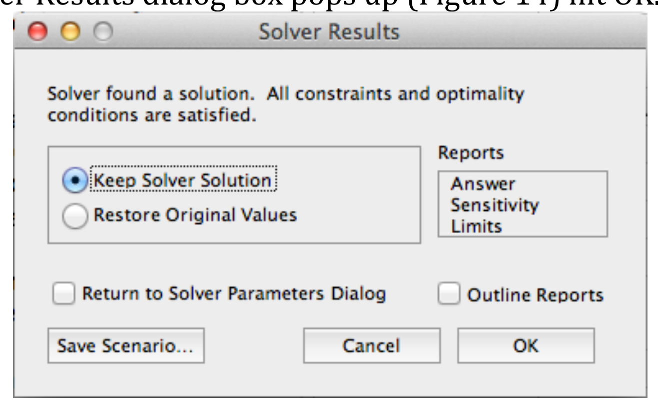

Figure 14: Solver Results dialog box.

4. Figure 12 shows the optimal policy for the example item.

10.5 The "LTD_CV", "LTD_L", and "LTD_V" Tabs

These three tabs implement some of the adjusted reduced models discussed in the report. The CV estimate model LTD_{CV} (Section 4.2.1) is implemented in "LTD_CV" tab. The mean inflation model LTD_{L} (Section 4.2.3) is implemented in "LTD_L" tab, although for computational reasons an approximation is used to update r^*_L and Q^*_L in each iteration instead of finding the exact solution. The variance inflation model LTD_V (Section 4.2.5) is implemented in "LTD_V" tab.

Each tab is laid out similarly to "LTD0" and contains a properly set-up solver model. The steps to use these tabs are the same as "LTD0".

DVs		
VMR	0.300	
Demand Distribution		
Mean	10	Items per day
Std dev	2	
Lead Time Distribution		
Mean	14.00	days per delivery
Std dev	2.049	
LTD Distribution		
Mean	140	items
Var	476	
Std dev	21.82	items
Alpha	41.18	
Beta	0.2941	

Policy Settings			**Solver Model**			
r'	171.1	items	$ 127.12			
Q'	14.49	items	2			
			TRUE			
Results	**MSRE**		TRUE			
Expected vs Best	4.63%		32767			
Realized vs Best	1.06%		0			
Expected vs Real	7.63%					
Constraints						
r >= -Q	-14.49					
Performance	Expected	SRE (Expecte	Realized	SRE (Realizec	SRE (Expecte	Units
On Hand	38.86	9.22%	40.45	7.57%	0.15%	items
Backorders	0.56	11.57%	2.14	234.92%	54.68%	items
Ready Rate	95%	0.00%	89%	0.46%	0.53%	
Order Frequency	0.690	2.02%	0.690	2.02%	0.00%	Orders per day
Safety Stock	31.06	11.20%	31.060	11.20%	0.00%	items
Expected Costs	Expected	SRE (Expecte	Realized	SRE (Realizec	SRE (Expecte	Units
Total Cost	$ 127.12	9.26%	$ 206.35	1.67%	14.74%	Dollars per day
Ordering Cost	$ 3.45	2.02%	$ 3.45	2.02%	0.00%	Dollars per day
Holding Cost	$ 97.16	9.22%	$ 101.12	7.57%	0.15%	Dollars per day
Backorder Cost	$ 26.52	11.57%	$ 101.79	234.92%	54.68%	Dollars per day
Value of On Hand Inventory	$ 388.62	9.22%	$ 404.47	7.57%	0.15%	Dollars
Safety Stock Value	$ 310.60	11.20%	$ 310.60	11.20%	0.00%	Dollars
Annual Relevant Cost	$ 36,721.47	8.65%	$ 38,167.44	7.09%	0.14%	Dollars per year

Figure 15: LTD_CV Screenshot with solution to example item.

DVs		**Percentile**	**Inflation Factor**	**Policy Parameters**			**Solver Model**			
L'	17.568	87.93%	1.255	r'	191.3		0.0002			
				Q'	6.49		1.0000			
							TRUE			
Demand Distribution				**Results**	**MSRE**		32767			
Mean	10	Items per day		Expected vs Best	23.56%		0			
Std dev	2.000			Realized vs Best	0.02%					
				Expected vs Real	23.96%					
Lead Time Distribution				**Constraint**						
Mean	17.57	days per delivery		Constraint	-6.49					
Std dev	0									
				Performance	Expected	SRE (Expect	Realized	SRE (Realized	SRE (Expectec	Units
LTD Distribution				On Hand	18.89	43.77%	55.36	0.01%	43.41%	items
Mean	175.7	items		Backorders	0.05	87.80%	0.84	0.00%	87.75%	items
Var	70.27			Ready Rate	98%	0.13%	95%	0.00%	0.13%	
Std dev	8.383	items		Order Frequency	1.541	240.22%	1.541	240.22%	0.00%	Orders per day
Alpha	439.2			Safety Stock	15.59	44.37%	51.271	0.96%	48.44%	items
Beta	2.5									
				Expected Costs	Expected	SRE (Expect	Realized	SRE (Realized	SRE (Expectec	Units
Problem Characteristics				Total Cost	$ 57.45	47.00%	$ 186.12	0.03%	47.79%	Dollars per day
Prob for r'	96.58%			Ordering Cost	$ 7.71	240.22%	$ 7.71	240.22%	0.00%	Dollars per day
r'	191.3			Holding Cost	$ 47.21	43.77%	$ 138.40	0.01%	43.41%	Dollars per day
Q'	6.489			Backorder Cost	$ 2.53	87.80%	$ 40.02	0.00%	87.75%	Dollars per day
				Value of On Hand Inventory	$ 188.86	43.77%	$ 553.58	0.01%	43.41%	Dollars
				Safety Stock Value	$ 155.88	44.37%	$ 512.71	0.96%	48.44%	Dollars
				Annual Relevant Cost	$ 20,045.92	37.78%	$ 53,326.77	0.06%	38.95%	Dollars per year

Figure 16: LTD_L screenshot with example item. Slightly different from results in Section 4.2.3.1 because implementation uses approximation.

A	B	C	G	H	I	J	K	L	M
DVs		**Inflation Factor**	**Policy Settings**			**Solver Model**			
Std D	8.264	4.1318	r'	186.7	items	$ 182.72			
			Q'	16.55	items	2			
						TRUE			
Demand Distribution			**Results**	**MSRE**		TRUE			
Mean	10	Items per day	Expected vs Best	0.00%		32767			
Std dev	8.264		Realized vs Best	0.00%		0			
			Expected vs Real	0.00%					
Lead Time Distribution			**Constraints**						
Mean	14.00	days per delivery	r >= -Q	-16.55					
Std dev	0								
			Performance	Expected	SRE (Expecte	Realized	SRE (Realizec	SRE (Expecte	Units
LTD Distribution			On Hand	55.80	0.00%	55.80	0.00%	0.00%	items
Mean	140.0	items	Backorders	0.85	0.00%	0.85	0.00%	0.00%	items
Var	956.00		Ready Rate	95%	0.00%	95%	0.00%	0.00%	
Std dev	30.919	items	Order Frequency	0.604	0.00%	0.604	0.00%	0.00%	Orders per day
Alpha	20.5		Safety Stock	46.69	0.00%	46.685	0.00%	0.00%	items
Beta	0.1464								
			Expected Costs	Expected	SRE (Expecte	Realized	SRE (Realizec	SRE (Expecte	Units
			Total Cost	$ 182.72	0.00%	$ 182.72	0.00%	0.00%	Dollars per day
			Ordering Cost	$ 3.02	0.00%	$ 3.02	0.00%	0.00%	Dollars per day
			Holding Cost	$ 139.51	0.00%	$ 139.51	0.00%	0.00%	Dollars per day
			Backorder Cost	$ 40.19	0.00%	$ 40.19	0.00%	0.00%	Dollars per day
			Value of On Hand Inventory	$ 558.04	0.00%	$ 558.04	0.00%	0.00%	Dollars
			Safety Stock Value	$ 466.85	0.00%	$ 466.85	0.00%	0.00%	Dollars
			Annual Relevant Cost	$ 52,024.41	0.00%	$ 52,024.41	0.00%	0.00%	Dollars per year

Figure 17: LTD_V screenshot showing solution to example item

10.6 THE "COMPARISONS" TAB

This tab compares all five LTD models implemented in this spreadsheet: LTD*, LTD_0, LTD_{CV}, LTD_L, and LTD_V. It is based on a PivotTable. Figure 18 shows how to compare the values of a single performance measure such as annual relevant cost across all five models.

Metric	Annual Relevant Cost					
Average of Value	Column Labels					
Row Labels	Expected	Realized	SRE(B)	SRE(E)	SRE(R)	Grand Total
LTD*	$ 52,024.70					$ 52,024.70
LTD0	$ 14,199.55	$ 20,541.13	9.531%	52.862%	36.623%	$ 6,948.33
LTD_CV	$ 36,721.47	$ 38,167.44	0.144%	8.653%	7.095%	$ 14,977.81
LTD_L	$ 20,045.92	$ 53,326.77	38.949%	37.784%	0.063%	$ 14,674.69
LTD_V	$ 52,024.41	$ 52,024.41	0.000%	0.000%	0.000%	$ 20,809.76
Grand Total	**$ 35,003.21**	**$ 41,014.94**	**12.156%**	**24.825%**	**10.945%**	**$ 16,146.56**

Figure 18: Comparing a single performance measure across all models

Several other metrics are also available. To examine a different measure of performance, choose from the drop-down box at top, as illustrated in Figure 19.

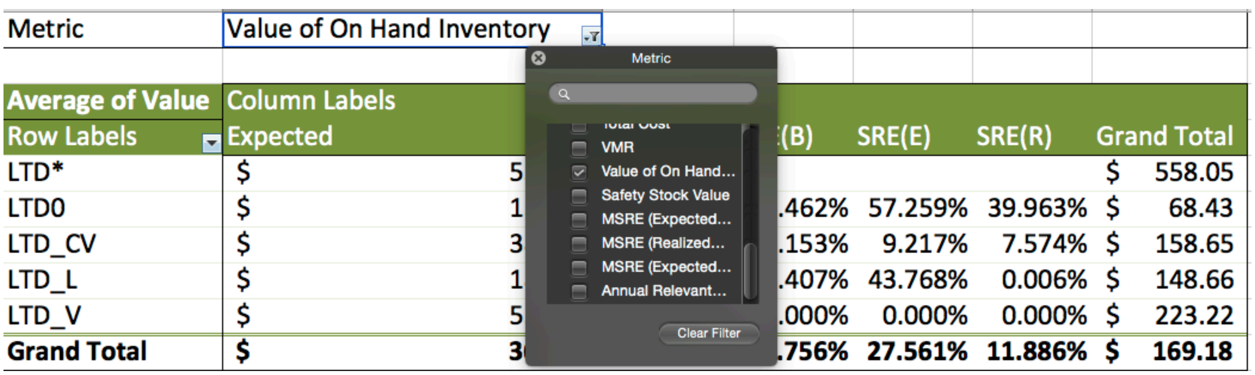

Figure 19: Choosing a performance measure

The Comparisons tab also includes several charts to visualize data. The data in the summary table (Figure 18) is presented in the chart immediately below it (Figure 20).

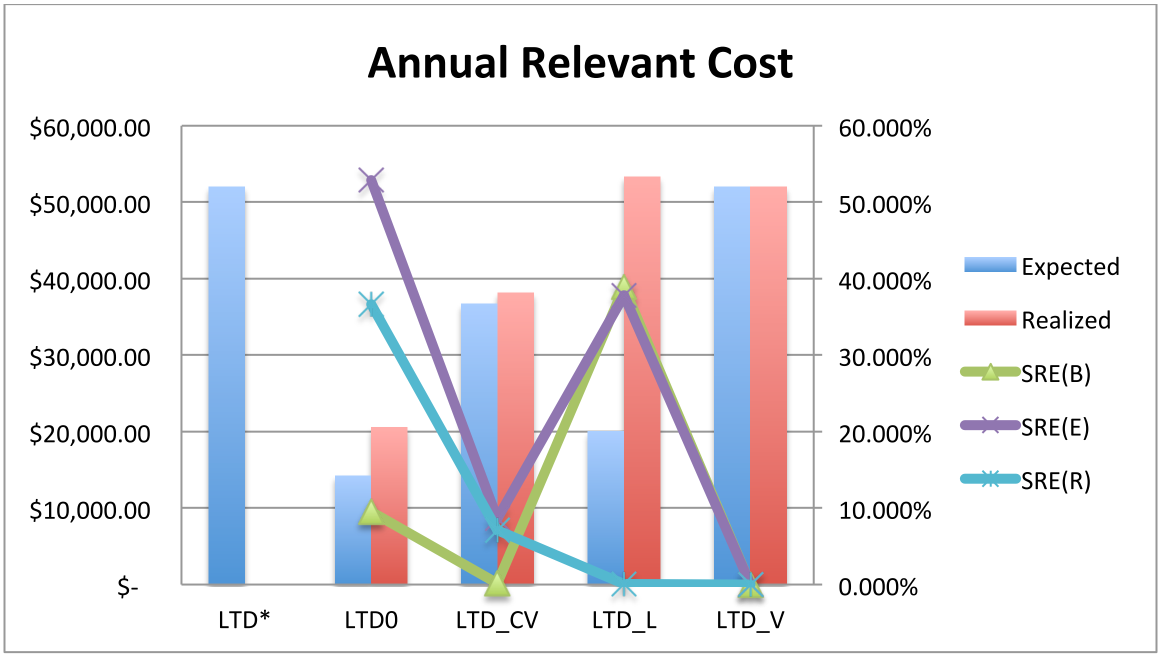

Figure 20: Chart comparing all 5 models

To the side of the table are some further charts. The top chart (Figure 21) shows the reorder point and reorder quantity for each model, allowing for comparisons of buffer stock and inventory position.

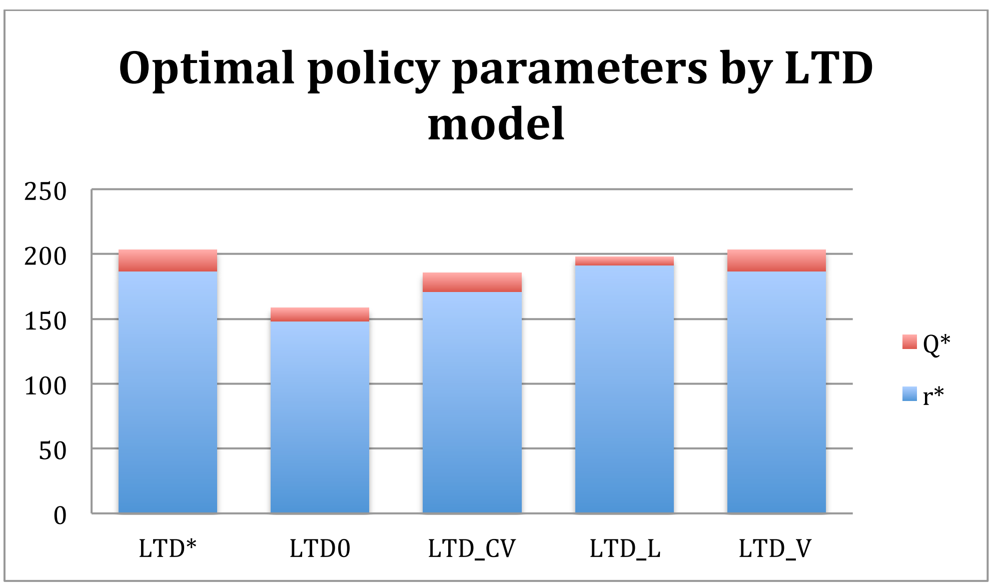

Figure 21: Policy Parameters

The bottom chart (Figure 22) allows comparing the LTD distributions produced by each of the LTD models. The full model, LTD*, is on the left. The bars depict the expected demand during replenishment lead time. It is notable that all models except LTD_L have the same mean demand. The error bars are the standard deviation of the LTD distribution from each model. Comparing them to LTD* shows how LTD_0, LTD_{CV}, and LTD_L underestimate the LTD variance, which is a problem as discussed in the body of the report.

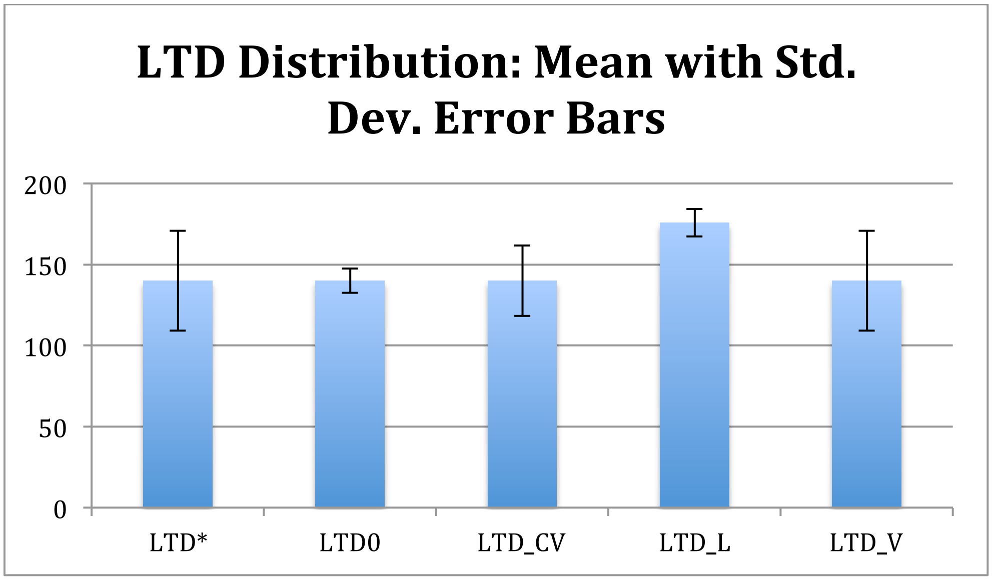

Figure 22: LTD distribution comparisons

10.7 The "Summary" Tab

For further processing, all the performance measures calculated in the worksheet can be exported by saving the "Summary" tab as a .csv file. Table 20 shows an excerpt from the worksheet. The relevant columns are "Model Name" (B), "Metric" (D), "Name" (F), and "Value" (H). The other columns are used internally. This sheet is protected to prevent accidental alterations. Data may be copied and sorted while protected.

Table 20: Example of Summary Tab

Sheet Name	Model Name	Abbr	Metric	Benchmark	Name	Address	Value
'Full LTD Model'	LTD*	star	r	Realized	Expected	rstar	186.7
'Full LTD Model'	LTD*	star	Q	Realized	Expected	Qstar	16.5
'Full LTD Model'	LTD*	star	r+Q	Realized	Expected	bigR	203.2
'Full LTD Model'	LTD*	LTD	mu	Realized	Expected	muLTD	140
'Full LTD Model'	LTD*	LTD	std	Realized	Expected	stdLTD	30.92
'Full LTD Model'	LTD*	star	Total Cost	Realized	Expected	Cstar	$182.72
'Full LTD Model'	LTD*	star	Ready Rate	Realized	Expected	Rrstar	95.0%

The workbook may be further developed and extended by changing the "Summary" and "Lookups" worksheets. These worksheets are protected to guard against accidental alterations, but the password for both sheets ("password") is provided in the workbook itself so that the user can develop the workbook further – for example, to deal with unexpected changes or add other metrics of interest.

11 Appendix C: Comprehensive List of Performance Measures in Prototype